Stability of river bed forms

M. Colombini *Università di Genova, DICCA, Genova, ITALY – col@dicca.unige.it*

ABSTRACT: Linear stability analyses provide a deep insight into the mechanisms that drive bed form formation, being able to identify regions in the space of the parameters where bed forms are expected to form and to show how the onset of instability is controlled by the relevant flow and sediment parameters. Moreover, the role of some parameters in inducing a transition between different types of bed forms can be made evident, as well as the possibility for different kinds of bed forms to coexist. By examining the results obtained in the framework of linear stability analysis, a unified view of the formation of free bed forms in straight channels is obtained.

1. INTRODUCTION

A variety of sediment waves develops from an initially flat bed when a uniform flow over an erodible bed is considered. They can be classified in terms of the geometric (e.g. wavelength, amplitude, shape) or kinematic (e.g. upstream or downstream propagation) characteristics of the bed form itself, in terms of the characteristics of the flow (e.g. subcritical or supercritical, hydraulically smooth or rough regime, amplitude and phase of the free-surface wave with respect to the bed wave) or of the sediments (e.g. finer or coarser material, bed load or suspended load). In general, more than one aspect is needed to mark the distinction. For these reasons, predicting, for any given set of the relevant flow and sediment parameters, which particular bed pattern will arise among the variety of possible, and often similar, configurations, is indeed a challenging task. Although very different scales are involved and several effects can be invoked to drive the instability process, yet a simple glance at the linearized form of the Exner continuity equation reveals that a single mechanism is ultimately responsible for the appearance of those periodic patterns we usually refer to as bed forms: the lag between the sediment transport and the bed topography, which controls both the growth and the celerity of the wave. Several effects contribute to this lag, bed evolution being ultimately due to a delicate balance between stabilizing and destabilizing effects.

2. BED FORM CLASSIFICATION

How can bed forms be told apart? How can one distinguish a dune from an antidune? Or a dune from a ripple? Or an alternate bar from an oblique dune? These questions may sound naive for the experimenter, who can often easily name a bed form just by looking at it, but are not so obvious for the theoretician, who, playing with different sets of the flow and sediment parameters, observes similar periodic patterns emerge. Moreover, though laboratory experiments are carefully designed to isolate a single type of bed form from the others, bed patterns often arise that do not easily fit in any of the above schematic categories.
River bed forms are commonly classified in terms of their characteristic longitudinal wavelengths. The distinction of bed patterns into micro-, meso- and macro-forms dates back to the first geomorphological observations (Allen, 1982) and

guided both experimental and theoretical researches in this field in the last century: nowadays, it is widely accepted that ripples scale with sediment grain size, dunes and antidunes with the flow depth, bars with the channel width. However, note that the word “scale” should be used with some caution: since the constant of proportionality can go from a few units (for bars) up to one thousand (for ripples), in this context “scale” does not represent an actual order of magnitude but, more appropriately, it should be considered as the most relevant physical length scale for that particular bed form.

Thus notwithstanding, the formation of river patterns is driven by the same basic mechanism so that such a neat separation among different bed forms should not always be expected to hold when the relevant flow and sediment parameters are varied continuously.

2.1. The case of dunes and ripples.

Let us consider differences and similarities between dunes and ripples. They both appear in the subcritical regime and propagate downstream, so that it is possible to discriminate ones from the others only in terms of their characteristic wavelengths, ripples typically being about one order of magnitude shorter than dunes.

It may be useful to report here the definition of dunes given by Guy, Simons & Richardson (1966), denoted in the following as GSR, in their remarkable experimental work: *“Dunes are bed features larger than ripples that are out of phase with any water-surface gravity waves that accompany them. Dunes generally form at larger flow and sediment transport rates than do ripples; however, ripples often form on the upstream slopes of dunes at smaller rates of flow”*. About half a century later, this distinction, albeit being evident to the experimenters, is still debated. Indeed, as Raudkivi (2006) writes: *“the change from ripples to dunes is terra incognita”*. When attempting to distinguish dunes from ripples, the only strong argument available remains that of the different wavelength, so that in the literature more than often the terms “ripple” and “dune” are interchanged, as if they were synonyms. Moreover, a variety of mixed terms (mega-ripples, micro-dunes, but also giant dunes and micro-ripples) have been coined.

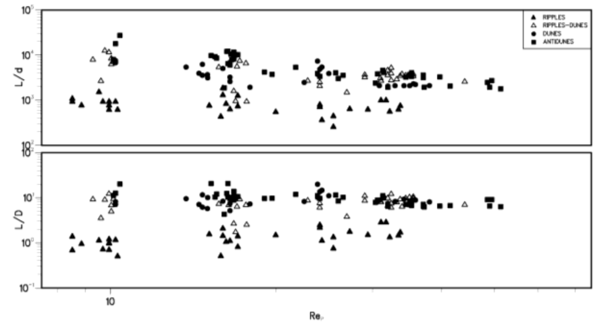

Figure 2.1: Experiments from GSR data sets: ripple and dune wavelengths as a function of Re_P; upper panel: scaled with the sediment grain size; lower panel: scaled with the flow depth.

In Figure 2.1, experimental data on dune and ripple wavelengths from the GSR dataset are presented, scaled with the sediment diameter d^* and with the flow depth D^* in the upper and lower panels, respectively.

The particle Reynolds number Re_P is defined as

$$Re_P = \frac{\sqrt{(s-1)gd^{*3}}}{\nu} \qquad (2.1)$$

where g is the gravitational acceleration, s the relative density of the sediment and ν is the kinematic viscosity of the fluid.

For the experimental range of grain sizes considered, ripple wavelengths appear to better correlate with the sediment diameter, since their ratio is found to attain an almost constant value of about 1000, as predicted by Yalin (1977), although a fairly large scatter is present. The opposite is true for dune wavelengths, which better scale with the flow depth, with a value of the ratio of about 10. It must be pointed out that, irrespective of the scaling adopted, in both panels the characteristic wavelengths for dunes and ripples are separated by about one decade.

Experimental observations reveal that dunes and ripples are morphologically similar, can coexist but are, indeed, distinct bed forms. Hence, a question remains unanswered: under which flow and sediment conditions should one expect to find ripples, or dunes or both? We will see in the following section that stability analyses can provide a hint in this regard, by relating ripple appearance to the transitional or smooth regime, whereas in the rough regime only dunes can form.

2.2. The case of alternate bars and oblique dunes.

The situation becomes even more complex when three-dimensionality is considered, since several spanwise modes can be identified, depending on the integer ratio between the channel width and the transverse half-wavelength. A typical example involves alternate and central bars, which are associated to the first and the second spanwise mode, respectively.

To investigate three-dimensional bed forms, the collection of flume experiments concerning dunes, antidunes and ripples of GSR, has been integrated with a data set, denoted as JSM, composed by the sand experimental runs extracted from Jaeggi (1984), Sukegawa (1971) and Muramoto and Fujita (1978). These runs have been selected among others since a clear distinction was made between alternate and diagonal bars, the latter being typically shorter than the former.

The distinction between diagonal and alternate bars was firstly proposed by Einstein and Shen (1964), who observed that diagonal bars are: *"a special case of the diagonal dune pattern, which occurs when the Froude number of the flow is nearly unity, at certain depth-to-width ratios. This pattern probably results from the water surface disturbance, since the diagonal bars oscillated transversely with the wave velocity of water depth and the entire bed pattern travels rapidly downstream with the flow."*.

In Figure 2.2, wavelength data of dunes, antidunes and bars extracted from the GSR and JSM data sets are presented. Data have been normalized with the half-width of the channel in the upper panel and with the flow depth in the lower panel. A comparison between the two panels reveals that bars do not scale properly with flow depth or, at least, they are more correlated with channel width. The opposite is true for dunes and antidunes.

As for the case of dunes and ripples, irrespective of the scale chosen to normalize, alternate bars are seen to be about an order of magnitude longer than dunes and antidunes. Diagonal bars seem to not fit with any of the scalings, their wavelengths falling just in between. Experimental observations suggest that diagonal bars can be considered as intermediate bed forms associated with the transition of dunes from two- to three-dimensional configurations.

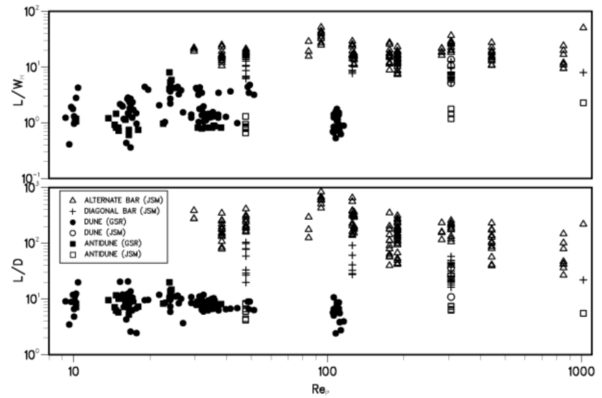

Figure 2.2: Experiments from GSR and JSM data sets: bars and dunes wavelengths as a function of the particle Reynolds number Re_P ; upper panel: scaled with the half-width of the channel; lower panel: scaled with flow depth.

Indeed, stability analysis confirms this suggestion, showing that diagonal bars represent the first transverse mode of instability of oblique dunes, which is morphologically similar to alternate bars. Alternatively, diagonal bars can be thought of as the results of the influence of the flow depth (and thus of the Froude number) on alternate bars.

3. THE PARAMETER SPACE

Bed form experimental measurements are usually collected at the end of the runs and are therefore relative to mature bed forms. To eliminate the effects of form resistance and of the sidewalls, an equivalent uniform flow is sought characterized by the same area velocity U^* and exerting the same shear stress on the bed, expressed in terms of the friction velocity u^*_f. These two quantities are related by a nondimensional Chézy coefficient C:

$$C = \frac{U}{u_f} = \sqrt{\frac{8}{f}} \qquad (3.1)$$

where f is the Darcy-Weisbach friction coefficient. On simple dimensional ground, the coefficient C can be shown to depend on two nondimensional parameters, both related to the bed roughness z^*_R:

$$C = C(z_R, Re_R) \qquad (3.2)$$

where z_R is the nondimensional roughness, which expresses the ratio between the bed roughness and the flow depth, and Re_R is the roughness Reynolds number, which expresses the ratio between the bed roughness and the thickness of the viscous sublayer. Moreover, it is customary to set the bed roughness equal to 2.5 times the grain diameter d^*, so that:

$$z_R = 2.5d \qquad Re_R = 2.5\frac{u_f^* d^*}{\nu} \qquad (3.3)$$

where d represents the nondimensional grain size. Several empirical relationships (ASCE, 1963; Cheng, 2008) are available for the determination of the function C of (3.2) in the different flow regimes. Typically, river flow pertains to the turbulent rough regime (Re_R>70) so that the role of Re_R can safely be neglected in the study of most bed forms. However, for relatively fine sediment and for relatively small values of the friction velocity (as is the case for ripples), flow can reach the transitional (5<Re_R<70) or the hydraulically smooth (Re_R<5) regime.

Among the relevant flow and sediment parameters, we recall the Froude number Fr and the Shields parameter θ.

$$Fr = \frac{U^*}{\sqrt{gD^*}} \qquad \vartheta = \frac{u_f^{*2}}{(s-1)gd^*} \qquad (3.4)$$

It is worth noting that the parameters just presented are interrelated. Hence, we can write, for example:

$$\vartheta = \frac{Fr^2}{(s-1)dC^2} \qquad Re_R = 2.5\sqrt{\vartheta}Re_P \qquad (3.5)$$

Hence, if hydraulically rough regime conditions are assumed, as it is customary for most river bed forms, only two parameters are to be set to identify the state of the system: one between the nondimensional sediment grain size d and the conductance Chézy coefficient C, which are related by (3.2), and one among the Froude number Fr and the Shields stress θ, which are related by (3.5).

If ripples have to be considered, then a third parameter must enter the analysis, namely a Reynolds number: either the particle Reynolds number Re_P or the roughness Reynolds number Re_R can be chosen, the two being related by (3.5).

4. BED FORM STABILITY

What is linear stability analysis all about?

Let us consider a system under steady equilibrium conditions and imagine to alter the state of this system (the so called 'base state') by slightly modifying the values of the variables and of the parameter that characterize the state of the system itself. If the perturbations are small enough, then the equations that govern the system can be suitably linearized around the base state and an eigenvalue problem is obtained, the solution of which provides information on the stability of the system.

In the study of river bed forms, the base flow is typically a uniform flow in an infinitely wide channel with active sediment transport. Perturbations, are assigned a specific spatial structure, which is periodic in the longitudinal streamwise direction, with wavenumber k_x,(dunes, antidunes, ripples), and in the transverse spanwise direction (alternate bars, oblique dunes), with wavenumber k_y. A specific temporal structure is assigned as well, which allows the perturbations to exponentially grow (decay) in amplitude, depending on the positive (negative) sign of the growth rate, as they propagate downstream (upstream) depending on the positive (negative) sign of the celerity.

Without entering into the details, it may be worthwhile at this point to outline the procedure that leads from the formulation of the problem to the actual evaluation of the growth rate and celerity.

First of all, the problem must be formulated, which means that a suitable set of equations has to be provided, whereby the dynamics of the flow-bed system is appropriately described. We can stick on generalities here by saying that the flow and the sediment transport models should be the simplest possible for the specific process one wants to simulate.

Focussing on the flow model, if we aim at describing the formation of dunes and antidunes, then the model should account for the interactions between the free-surface and the bed, so that we can distinguish between sub- and super-critical bed forms. Hydraulically rough conditions can be safely assumed. On the other hand, if the goal is the modelling of ripples, the free-surface effect can be disregarded, but the model should cover the smooth and transitional regimes, since this appears to be the characteristic feature of ripples. Double-periodic bed forms like alternate bars or oblique dunes would in principle require a three-dimensional flow model, even though simpler depth-averaged models can be sufficient to describe the flow if the longitudinal and transverse wavelengths are large enough with respect to the flow depth so as to satisfy the shallow-water approximation, as is the case for bars.

The minimal ingredients for the sediment transport model are the Exner equation and a suitable relationship that provides the amount of sediment moving as bedload in terms of the Shields stress.

For any given set of the flow and sediment transport parameters, the solution of the eigenvalue problem obtained after linearization provides the growth rate and celerity of the bed perturbations as a function of the wavenumber(s). Stability plots can be constructed showing the ranges of unstable (positive growth rate) and stable (negative growth rate) wavenumbers. Marginal (vanishing growth rate) curves mark the borders between stable and unstable regions. Moreover, the wavenumber of maximum growth rate identifies the wavelengths that are likely to be selected by the instability process.

The stability plot of in Figure 4.1 is relevant for the study of dunes and antidunes (Colombini, 2004). The shaded areas represent the unstable regions for dunes (lower area) and antidunes (upper area).

Let us perform an ideal laboratory experiment, whereby we want to investigate the transition from dunes to antidunes observed as the Froude number is increased. Moreover, we want to keep the flow depth constant, so as to maintain the same value of the nondimensional grain size *d* and thus of *C*. This can be accomplished by letting the discharge increase, but note that, in order to keep the depth constant, the average slope of the channel must increase as well.

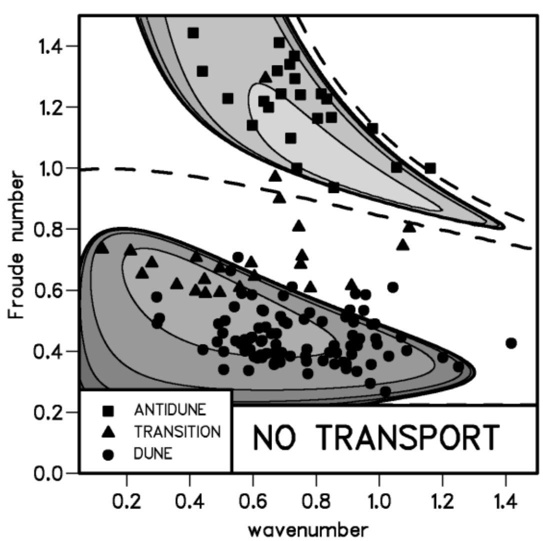

Figure 4.1: Stability plots showing the region of instability for dunes and antidunes. Markers represent experimental data from GSR.

By doing so, we move vertically along the plot of Figure 4.1, which has been obtained for a fixed value of the conductance coefficient *C*.

In our experiment, starting from very low values of the Froude number, we cross, in turn, the no-transport boundary (where sediments start to move), the dune region (where dunes are unstable), the upper plane bed region (where neither dunes nor antidunes form) and, finally, the antidune region (where antidunes are unstable).

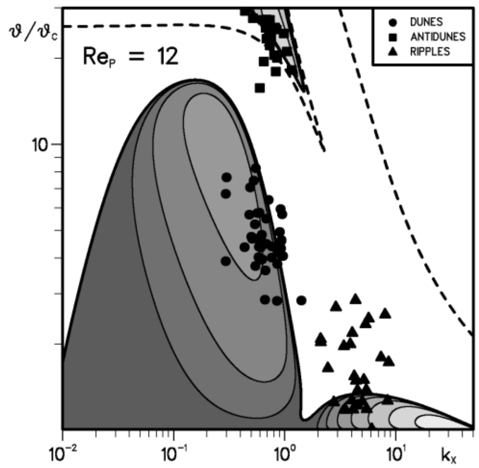

Figure 4.2: Stability plot showing the region of instability for dunes and ripples. Markers represent experimental data from GSR.

The stability plot of Figure 4.2 is relevant for the study of ripple formation (Colombini & Stocchino, 2011). The value of Re_P has been chosen small enough for the flow to be in the hydraulically smooth regime when the Shields number is close to the threshold for sediment motion. For relatively small values of θ, two maxima of the growth rate are observed, one characterized by a wavenumber (scaled with flow depth) of O(1), which represents dunes, the other of O(10), which represents ripples. The presence of two maxima indicates that both ripples and dune are simultaneously unstable and compete. If a higher Re_P (i.e. a coarser material) is used, the unstable region on the right disappears and only dunes are predicted to form.

Finally, although this result cannot be illustrated by means of a single plot, a three-dimensional stability analysis is able to shed some light on the transition from two-dimensional dunes to alternate bars observed as the flow gets shallower, which includes the formation of diagonal bars as intermediate bed forms (Colombini & Stocchino, 2012).

5. CONCLUSIONS

Linear stability analysis provides a deep insight on the mechanisms that drive the formation of bed forms and on the parameter that controls the onset of the instability, together with an estimate of the most unstable wavelength, the one that will likely be selected by the instability process.

Since one of the outputs of a linear stability analysis is the regime (i.e. the region in the parameter space) where some particular bed form is expected to appear, bed form regime diagrams can be built, providing theoretical support to the analogous experimental diagrams, which guided the research on bed forms in the last decades (e.g. Vanoni, 1974). Moreover, linear stability allows to investigate the boundaries of such regions, where transition from one bed form to another takes place, a situation quite difficult to be examined experimentally.

Within the same linear framework, flow and sediment transport models can be tailored to suit the needs of the specific problem at hand. Suspended load, whereby sediment are not anymore confined in a thin layer close to the bed, can be accounted for by adding an additional equation for the sediment concentration along the vertical. Sorting, which involves bedload transport of an heterogeneous mixture of sediment of different sizes, can be included by approximating the continuous grain size distribution with a discrete number of fractions and introducing, for each fraction, a suitable Exner-like equations.

Although all the examples cited above, as well as my personal experience, are limited to river bed forms, the same techniques can be (and have been) applied to study the formation of bed forms in tidal, estuarine, and coastal environments, by simply replacing the steady uniform flow that defines the base state in river stability analysis with a suitable combination of waves and currents.

The formulation of the eigenvalue problem and the techniques adopted for its solution may look a little different, but the similarities are usually more than the differences, so that analogies can be cast between bed forms appearing in the different environments.

Aeolian bed form stability can be studied as well, but care must be taken in the modelling of sediment transport, since the sediment dynamics is strongly affected by the media.

Finally, we can certainly conclude that stability analysis has taught us a lot about the formation of bed forms. Will it be able to teach us more in the future? Or will it succumb under the weight of the increasing number of sophisticated numerical models that mimics nature so well to make the resulting simulations undistinguishable from the real thing?

A tough question, indeed.

6. REFERENCES

Allen, J. 1982. Sedimentary Structures: Their Character and Physical Basis - Vol. 1. Elsevier, Amsterdam.

ASCE, T. C. 1963. Friction factors in open channels. J. Hydraulic Div., 89 (HY2):97–143.

Cheng, N.-S. 2008. Formulas for friction factor in transitional regimes. J. Hydraulic Engng., 134:1357–1362.

Colombini, M. 2004. Revisiting the linear theory of sand dune formation. J. Fluid Mech., 502:1–16.

Colombini, M. and Stocchino, A. 2011. Ripple and dune formation in rivers. J. Fluid Mech., 673:121–131.

Colombini, M. and Stocchino, A. 2012. Three-dimensional river bed forms. J. Fluid Mech., 695:73–80.

Einstein, H. and Shen, H. 1964. A study on meandering in straight alluvial channels. J. Geophys. Res., 69:5239–5247.

Guy, H. P., Simons, D. B., and Richardson, E. V. 1966. Summary of alluvial channel data from flume experiments 1956-61. Prof. paper 462-I, U.S. Geol. Survey.

Jaeggi, M. 1984. Formation and effects of alternate bars. J. Hydraulic Engng., 110:142–156.

Muramoto, Y. and Fujita, Y. 1978. The classification of meso-scale river bed configuration and the criterion of its formation. In Proc. 22nd Japanese Conf. on Hydraulics, pages 275–282, Japan.

Raudkivi, A. 2006. Transition from ripples to dunes. J. Hydraulic Engng., 132:1316–1320.

Sukegawa, N. 1971. Study on meandering of streams in straight channels. Technical report, Bureau of Resources, Dept. Science & Technology, Japan.

Vanoni, V. A. 1974. Factors determining bed forms of alluvial streams. Journal of the Hydraulics Division, 100(3):363–377.

Yalin, M. S. 1977. On the determination of ripple length. J. Hydraulic Div. ASCE, 103:439–44