

## Three-dimensional spatial variations of suspended sediment concentration over vortex ripples

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### Abstract

A three-dimensional mixture theory model (SedMix3D) was used to simulate the flow and sediment transport over vortex ripples for scaled laboratory conditions. SedMix3D treats the fluid-sediment mixture as a continuum of varying density and viscosity with the concentration of sediment and velocity of the mixture calculated using a sediment flux equation coupled to the Navier-Stokes equations for the mixture. Mixture theory allows the model to simulate the three-dimensional flow and sediment concentration within and above an evolving sediment bed. Grid spacing was on the order of a sediment grain diameter and time steps were  $O(10^{-5}$  s). The simulation was forced with a time series of free-stream velocity measured in a free-surface laboratory flume. Spatial variations in the simulated suspended sediment concentration were primarily associated with the non-uniform generation of vortex structures over the ripple flanks. The suspended sediment was initially picked up in regions of high vorticity, and then caused a damping of the vorticity while being advected through the water column.

## 1. INTRODUCTION

Bedforms on the seafloor in the neashore coastal and continental shelf regions impact erosion and deposition, waves and currents, and coastal infrastructure. Ripples are ubiquitous in sandy coastal regions, generally covering the seafloor in depths up to 20 m during temperate weather (i.e. during non-extreme wave events). Ripples drastically affect the total sediment transport and wave energy dissipation; however, it is difficult to quantify their total effect due to the complexity that ripples bring to the hydrodynamics of bottom boundary layer flow.

The hydrodynamics above sand ripples are dominated by the coherent vortices formed on the ripple flanks and ejected into the water column at flow reversal. This process erodes sediment off the surface of the ripple and suspends it into the water column where it is then advected with the flow (Thorne et al., 2003, van der Werf et al., 2006).

Recent advances in high performance computing hardware and software have allowed us to implement three-dimensional, high-resolution simulations to study small-scale bedform dynamics. Experimental (laboratory and field) and complex numerical models are all necessary to better understand and quantify the effect of ripples on sediment transport and the turbulent dynamics of the wave boundary layer. However, most research examining sediment transport over three-dimensional dynamic beds has been limited to laboratory and field studies. Recent advances have allowed for the measurement of vortex dynamics and sediment transport in the laboratory (van der Werf, 2006) and field (Traykovski, 1999; Hurther and Thorne, 2011). Few three-dimensional, high-resolution models exist to study small-scale sediment dynamics in detail. Here we use a mixture theory model to study the dynamic coupling of vorticity and suspended sediment and their spatial variability over dynamic sandy rippled beds.

## 2. METHODOLOGY

### 2.1. Numerical model

Using the three-dimensional Navier-Stokes solver, SedMix3D, we performed a high spatial and temporal resolution simulation of oscillatory flow over a sandy rippled bed for 18 wave periods.

The model equations are based on the mixture velocity (the velocity of the sediment and fluid as a continuum) as defined by mixture theory. SedMix3D is a one-phase continuum model, with a scalar quantity of sediment concentration that determines the mixture's bulk properties (density, viscosity, settling velocity) at every grid point (Penko et al., 2011). The mixture is treated as a continuum of varying density and viscosity with the concentration of sediment and velocity of the mixture calculated with a sediment flux equation coupled with the Navier-Stokes equations for the mixture. The mixture continuity equation was derived by combining the fluid and sediment phase continuity equations,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (1)$$

where  $\mathbf{u}$  is the mixture velocity and  $\rho$  is the mixture density,

$$\rho = \phi \rho_s + (1 - \phi) \rho_f \quad (2)$$

where  $\phi$  is the sediment volumetric concentration, and  $\rho_s$  and  $\rho_f$  are the sediment and fluid densities, respectively.

The mixture momentum equation was derived from the sum of the fluid and sediment phase momentum equations,

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \nabla \cdot (\mu \nabla \mathbf{u}) + \mathbf{F} - \rho \mathbf{g} - S_b \mathbf{u} \quad (3)$$

where  $P$  is the mixture pressure,  $\mu$  is the effective viscosity,  $\mathbf{F}$  is the external driving force vector per unit volume,  $\mathbf{g}$  is gravitational acceleration (981 cm/s<sup>2</sup>), and  $S_b$  is the particle pressure parameterization.

SedMix3D employs a modified Eilers equation (Eilers, 1941) to represent effective viscosity,  $\mu$ , here scaled by the pure water viscosity,  $\mu_f$ ,

$$\frac{\mu}{\mu_f} = \left[ 1 + \frac{0.5[\mu]\phi}{1 - \phi/\phi_m} \right]^2 \quad (4)$$

where  $[\mu]$  is the intrinsic viscosity, a dimensionless parameter representing the sediment grain shape, and  $0.0 < \phi < 0.63$ , with the lower bound representing pure water and upper bound roughly corresponding to the maximum concentration of a packed sediment bed.

The concentration of sediment is modeled with a sediment flux equation (Nir and Acrivos, 1990) that balances the temporal gradients in sediment concentration with advection, gravity, and shear-induced diffusion,

$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = D \nabla^2 \phi - \frac{\partial \phi W_t}{\partial z} \quad (5)$$

where  $W_t$  is the concentration dependent settling velocity (Richardson and Zaki, 1954) and  $Re_p$  is the particle Reynolds number. The shear-induced diffusion of sediment,  $D$ , is a function of grain size, volumetric concentration, and mixture stresses (Leighton and Acrivos, 1986),

$$D = \frac{1}{4} d^2 \beta(\phi) |\nabla \mathbf{u}| = 0 \quad (6)$$

and where  $d$  is the grain size diameter,  $\alpha$  is an empirical constant, and  $\beta$  is an empirical function of concentration. Previously, SedMix3D demonstrated good agreement with detailed laboratory observations using particle image velocimetry to measure vortex dynamics over rippled beds with uniform grain size (Penko et al., 2013). Here, we initialized the numerical simulation with the measured bed profile and imposed hydrodynamic conditions.

### 2.2. Simulation set-up

The simulation had a domain size of 19.0 cm x 2.4 cm x 14.2 cm with approximately 1.5 million grid points and ran on 96 processors on a Department of Defense high performance computer. The 36 s real-time simulation took approximately 170 wall-clock hours to complete. The laboratory data used to initialize and force the model simulation were collected using Particle Image Velocimetry (PIV) in a free-surface wave flume with a wave generator that produced regular sinusoidal waves 5 cm in height with 2 s periods over an artificial sediment bed.

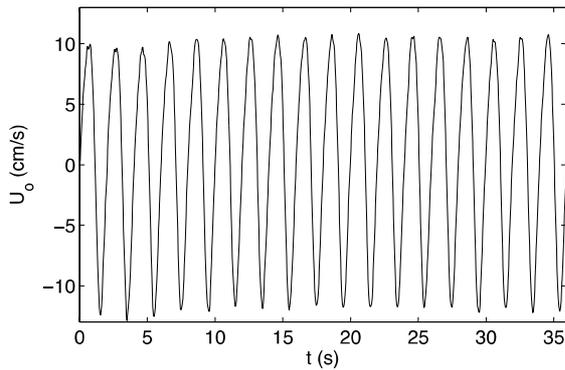


Figure 1. Free stream velocity time series recorded by an ADV in a free surface flume that was used as forcing in the simulations.

Two-dimensional ripples persisted during the experiment with wavelengths of  $\sim 5$  cm and heights of  $\sim 2$  cm. The simulation was forced with a time series of free stream velocity recorded with an ADV (Figure 1) for a total of 18 wave periods (36 s). The first 9 wave periods of the simulation were discarded for model spin-up and not included in the data analysis.

The initial bed consisted of four ripples (height  $\sim 2$  cm, length  $\sim 5$  cm) extracted from profiles measured in the laboratory (Figure 2). The initial bed profile was determined from the measured laboratory bed profile at the beginning of the experiment and then evolved with the oscillatory flow.

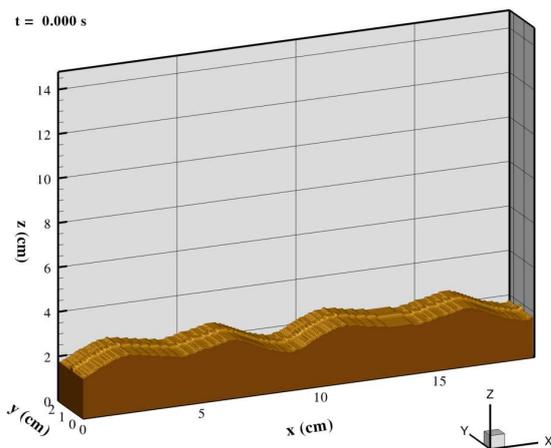


Figure 2. The initial simulation bed profile in the simulation coordinate system.

The bed profile reached equilibrium in the simulation by the seventh wave period and the ripples remained in equilibrium for the remainder of the simulation. Due to a lengthy model spin-up time, only the last 9 wave periods were used in the analysis. The model output of velocity and concentration was ensemble-averaged over the 9 wave periods and the results are presented in the following section.

### 3. RESULTS

SedMix3D outputs the three-component velocity vector and volumetric concentration of sediment ( $0 < \phi < 0.63$ ) at every grid point within the domain. The mixture theory approach is advantageous for examining sand ripple dynamics because the model simulates the velocity and concentration everywhere, from the top of the boundary layer to the packed sand bed, including the highly concentrated bedload layer that is very difficult to measure in situ and arbitrary in thickness. One can also directly examine the dynamically coupled effect of the suspended sediment on the vorticity and conversely, the vorticity on the transport of sediment.

A plan view (x-y) of the vorticity magnitude and the suspended sediment concentration contour levels for  $\phi = 0.01$  (blue solid line) and  $\phi = 0.1$  (magenta dashed line) at an elevation of  $z = 2.86$  cm is plotted in Figure 3. The insets in the upper left corners denote the phase of the wave for each subfigure. Note the significant variability of the vorticity and bursts of suspended sediment in the cross-flow (y) direction (Figure 3a). Immediately before flow reversal, sediment was picked up in the regions of strong vorticity (Figure 3a). Once suspended in the water column, the sediment began to damp the vorticity, causing pockets of decreased vorticity and consequently causing a spatial variability in the vortex dissipation (Figure 3b). As the flow began to accelerate, the model produced strong and spatially uniform vortex structures over ripple crests (Figure 3c). However, over the troughs, where the flow decelerated, vortices broke-up and dissipated at random, producing more three-dimensional variability than in the accelerated flow over the crests (Figure

3a,b). Also just after flow reversal, the suspended sediment concentrated in the leading (left) edge of the ejected vortex, damping out most of the vorticity before maximum flow (Figure 3b,c).

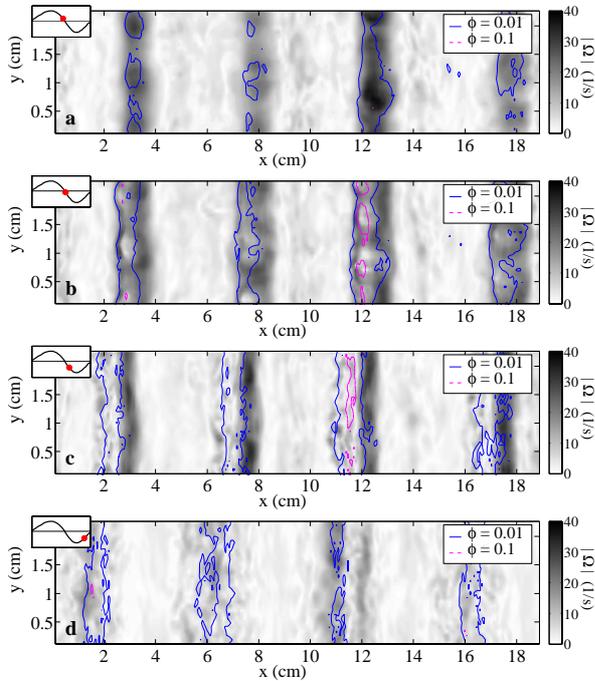


Figure 3. Plan view of the ensemble-averaged vorticity magnitude (filled black and white contours) and suspended sediment concentration (blue solid and magenta dashed contour lines) at  $z = 2.86$  cm. Insets show the ensemble-averaged free stream velocity (black line) and the position of the wave for each figure (red dot).

The simulation also showed a suspended sediment burst on the flank of the ripples after maximum flow, despite relatively low vorticity, which is a behavior we have observed in laboratory experiments of oscillatory flow over sand ripples (Figure 3d).

The cross-flow variability is also evident in a plot of the three-dimensional swirling strength (Figure 4). Swirling strength is a scalar quantity of the vorticity that identifies the closed rotational vortex structures in turbulent flow (Zhou et al., 1999). The method is effective in determining the location of vortex cores by excluding the vorticity due to boundary-generated shear, but does not identify

the rotational direction. At flow reversal (Figure 4a), the vortices over the troughs were very three-dimensional with high spatial variability, especially in the cross-flow direction. However, where the flow accelerated over the ripple crests, the vortices were more uniform, with very little variability in the cross-flow direction. After maximum flow (Figure 4b), the vortices are stretched and dissipated by the strong flow. The vortices dissipate randomly, especially over the ripple troughs.

#### 4. CONCLUSIONS

To the authors' knowledge, no one previously has been able to simulate the three-dimensionality of small-scale boundary layer processes in this amount of detail. SedMix3D allows for detailed examinations of the three-dimensional boundary layer processes occurring due to oscillatory flow over sand ripples. Here, we present three-dimensional model results of vorticity and suspended sediment concentrations throughout the phase of a wave. Over ripple crests and during the accelerated portion of the flow the model produced strong uniform vortex structures. However, over the troughs, where the flow decelerated, vortices broke-up and dissipated at random, producing more three-dimensional variability than in the accelerated flow over the crests. Spatial variations in the simulated suspended sediment concentration were primarily associated with the non-uniform generation of vortex structures over the ripple flanks and non-uniform vortex dissipation as flow accelerated. The suspended sediment was initially picked up in regions of high vorticity, and then damped the vorticity while being advected through the water column. The simulation also showed bursts of suspended sediment concentration when vorticity was low, similar to observations of laboratory experiments.

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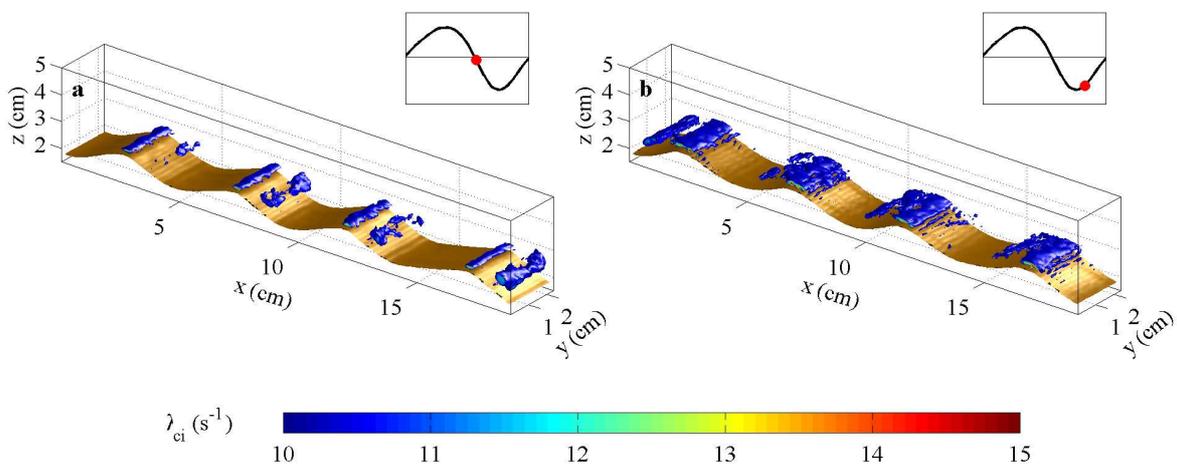


Figure 4. The plot shows the ensemble-averaged isosurface of the three-dimensional swirling strength at two phases of the wave. The blue surface indicates a swirling strength of  $10 \text{ s}^{-1}$  and the color contours represent the swirling strength inside a vortex that intersects the edge of the domain. The brown isosurface represents the instantaneous bed at 57% concentration by volume.

