Ripples and dunes in a turbulent stream

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ABSTRACT: It is widely accepted that both ripples and dunes form in rivers by primary linear instability, the wavelength of the former scaling on the grain size, that of the latter being controlled by the water depth. We revisit here this problem, starting from the derivation of the turbulent flow over a wavy bottom, with or without a free surface. We show that the presence of a free surface always has a stabilising effect, whatever the choice of the turbulence closure and of the sand transport model. Consequently, dunes cannot result from a primary instability. We propose a weakly non-linear description suggesting that dunes result from a pattern coarsening process, the final wavelength being that for which the bedform amplitude is maximum.

1 INTRODUCTION

In rivers, ripples and dunes are patterns at two well-separated wavelengths λ, that are respectively controlled by the grain size d and the depth of water H (Allen, 1980). Since the pioneering work of (Kennedy, 1963), it is commonly accepted that both patterns form by a primary linear instability (Colombini, 2004), statement that we wish to revisit in this paper.

Because the hydrodynamic timescales are much shorter than those of the bedform growth, the understanding of pattern formation in rivers can be split into a hydrodynamic problem and another regarding sediment transport issues. We present here the general mechanism responsible for the instability of a flat sand bed. Emphasizing the stabilizing role of the free surface, we evidence that river dunes can not form from a linear instability. Finally, we discuss a non-linear criterion for the selection of the size of dunes and mega-dunes.

2 A VERY GENERAL FRAMEWORK

2.1 *A purely hydrodynamic problem*

In this context, we compute the two dimensional turbulent flow confined between a free surface and a periodic wavy bottom. This bottom is characterized by its wavelength λ, or its wave-number $k=2\pi/\lambda$ and its amplitude 2ζ (see Fig. 1). The dynamical equations governing the evolution of the mean flow are closed by the means of a Prandtl mixing length approach, which relates the stress Reynolds tensor to the velocity gradients. The equations are expanded with respect to the bottom corrugation aspect ratio $k\zeta$ which is the small parameter of the problem.

A main output of this hydrodynamic calculation is the phase difference between the bottom topography $Z=\zeta e^{ikx}$ and the basal shear stress τ. We introduce two dimensionless functions A and B, defined by:

$$\tau = \tau_0 [A(k) + iB(k)] k\zeta e^{ikx} \quad (1)$$

where τ_0 is the reference basal shear stress over a flat bed. A represents the component of the stress tensor in phase with the bottom topography and B controls the phase shift. Its sign gives the position of the shear stress maximum with respect to the crest of the bumps. For instance, when $B>0$, the maximum of τ is shifted upstream. This generic case is represented in Figure 1. The location of shear maximum corresponds to the position where the streamlines are squeezed. On the other hand, a negative value of B corresponds to a downstream shift of the maximum of τ. Except the hydrodynamic roughness z_0, turbulence does not introduce any characteristic length scales, so that, the spatial shift is proportional to λ and the ratio B/A.

These coefficients A and B show weak (logarithmic) dependencies with kz_0, and their typical values are around unity. We have checked that more sophisticated descriptions of the turbulence, e.g. stress anisotropy or second order turbulent closure, do not alter much these functions. However, a potential des-

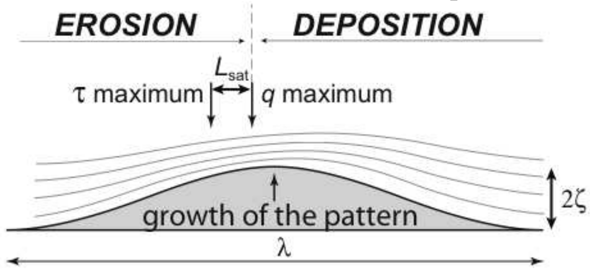

Figure 1. Sketch of the instability of a wavy bottom sheared by a turbulent flow.

cription of the flow does not lead to any phase shift between Z and τ.

2.2 *The sediment transport issue*

We now consider an erodible bottom sheared by a turbulent stream. If the strength of the flow is sufficiently high (above a threshold τ_{th}), grains get detached from the bed. In turn, the moving grains interact with the flow and decelerate it, so that the flow can only erode a limited quantity of sediment. We call $q_{sat}(\tau)$, the saturated flux, which corresponds to the quantity of sediment that can be transported by the flow at equilibrium.

The equilibrium state is not immediately reached and the relaxation towards this equilibrium depends on different processes limiting sediment transport, e.g. the ejection of the grains, the grain/fluid inertia. The choice of the relevant processes that have to be taken into account to describe the sediment transport is still a matter of debate and depends on the regime of the flow. In any case, the dominant process is the slowest of them and we call L_{sat} the spatial lag between the sand flux q and its saturated value q_{sat}. In other words, L_{sat} and q_{sat} integrate all details of sediment transport we need for our purpose.

3 LINEAR ANALYSIS

3.1 *The instability mechanism*

We are now able to describe the mechanism, which makes an erodible flat bed unstable. Consider a bump such as that in Figure 1. It will grow if its crest is in the deposition zone. Following the mass conservation relation, the position of the flux maximum separates the erosion zone upstream from the downstream zone of accretion. The criterion of instability can then be geometrically described as follows. Starting from the crest, the shear stress and thus the saturated flux is maximum at a distance $\sim\lambda B/A$ from it. The actual flux maximum is found after a space lag L_{sat} (see Fig. 1).

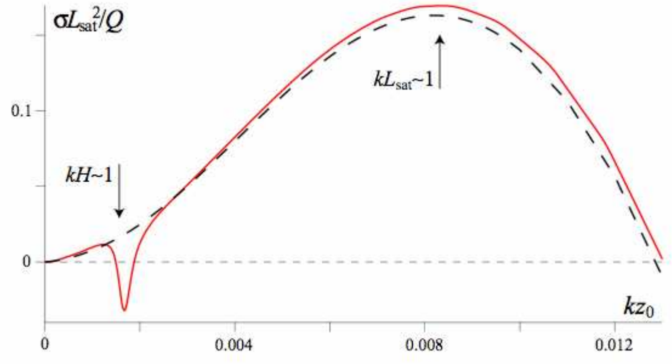

Figure 2. Dimensionless growth rate as a function of the wave number k adimensionnalised by the hydrodynamic roughness z_0. The dashed line corresponds to an asymptotically large flow depth and the solid line is for a finite flow depth H (here for Fr=0.8).

Within a linear analysis of the flow, these qualitative arguments can be translated into the mathematical expression of the dispersion relation, which expresses the growth rate σ as a function of the wavenumber k. In the case of the shear velocity much larger than the transport threshold, one gets:

$$\sigma = Qk^2 \frac{B - AkL_{sat}}{1 + (kL_{sat})^2} \qquad (2)$$

where Q is the reference saturated flux over a flat sand bed. σ is positive for large wave-lengths, which are thus unstable. On the other hand, bumps with small λ are stable (σ<0). This relation is plotted in Figure 2. It shows a pronounced maximum for kL_{sat}~1. This length-scale is the size at which ripple pattern first appears.

3.2 *Effect of the free surface*

The presence of the free surface modifies the shape of the dispersion relation. In comparison to the previous case, a stabilized zone (σ<0) appears for wavelengths scaling on the flow depth (see solid line in Fig. 2). This dip is more pronounced for larger Froude numbers.

This stabilization is due to a resonance phenomenon. The periodic wavy bottom excites surface waves at the wavelength λ. The phase between the two varies from 0 to π, so that, at the resonance the streamlines are squeezed downstream the crests (see Fig. 3). In conclusion, the growth rate is always lower with than without a free surface. Therefore, river dunes can not form by a linear instability of the bed.

4 WEAKLY NON-LINEAR RESULTS

To go beyond the linear analysis, we have extended the expansion of the flow field up to the third

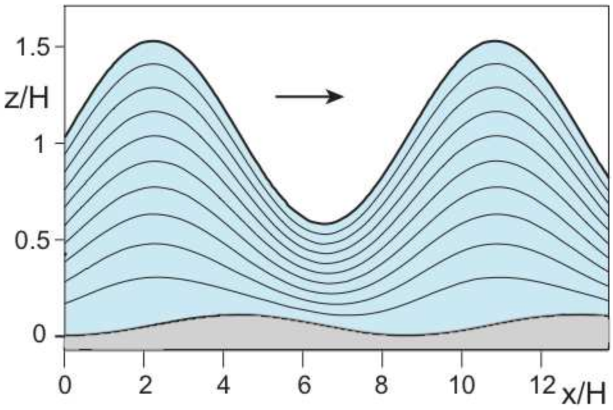

Figure 3. Sketch of the resonance of the free surface: the streamlines are squeezed downstream the crests of the wavy bottom. This is a stabilizing mechanism.

order in the aspect ratio ζ/λ, to get the first non-linear corrections to the basal shear stress.

The first result of these calculations is that the phase shift between *Z* and τ decreases when ζ increases. For a given λ, we call ζ_s the amplitude for which the position of the maximum of *q* precisely coincides with the crest of the bed corrugation, which corresponds to a steady propagative pattern. ζ_s is represented as a function of λ in Figure 4. In the case of an infinite turbulent boundary layer, the relationship between ζ_s and λ is linear, which corresponds to a constant aspect ratio, whose value (2ζ□/λ≈0.07) matches with observations.

In the presence of the free surface, the streamlines become compressed over the crests, causing the steady amplitudes to converge to smaller values. Moreover, the stabilizing mechanism from resonance amplifies this effect. The resulting relation between ζ_s and λ presents a pronounced maximum around λ=*H*, remarkably insensitive to variations of parameters of the modeling (see Fig. 4). A drift along the□□□□□ ζ_s(λ □□□□□□ponds to a pattern coarsening, i.e. an increase of the wavelength over time. We expect this coarsening to stop at the maximum for theoretical reasons (Politi & Misbah, 2004). Besides, the largest ζ_s is the most likely to dominate the flow and to be visualized. The physical mechanisms involved in the pattern coarsening (e.g. ripples collisions and interactions) are still to be clearly identified and need to be studied in more details in this context.

Finally, we can notice in Figure 4 a second smoother maximum for greater wavelengths. In contrast with the previous one, its position and amplitude are more sensitive to the parameters. For instance, its amplitude decreases for larger Froude numbers. Although these preliminary results require further work, this second peak suggests an interesting way to explain the formation of mega-dunes in large rivers, studied for example in Rio Paraná (Parsons & al, 2005).

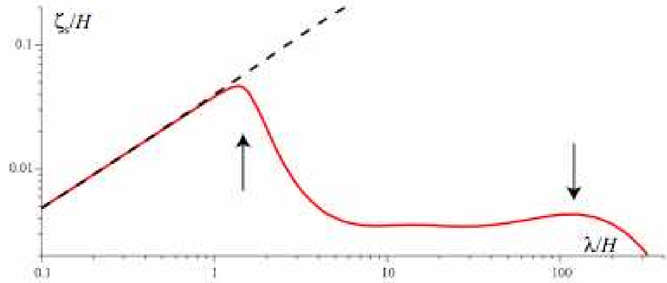

Figure 4. Amplitude of the steady propagative dune as a function of its wavelength (both normalized by *H*). The dashed line corresponds to an asymptotically large flow depth and the solid line is for a finite flow depth *H* (here for Fr=0.3).

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