On the development of a bedform migration model

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ABSTRACT: A model linking subaqueous dune migration to the effective (grain related) shear stress is calibrated by means of flume data for bedform dimensions and migration rates. The effective shear stress is calculated on the basis of a new method assuming a near-bed boundary layer above the mean bed level in which the current velocity accelerates towards the bedform top. As a consequence, the effective bed shear stress corresponds to the shear stress acting directly on top of the bedform. The model operates with critical Shields stresses as function of the grain Reynolds number. It predicts the deposition (volume per unit time and width) of natural packed bed material on the bedform lee side, qb_{lee}. The model is simple, built on a rational description of simplified sediment mechanics, and its calibration constant can be explained in accordance with estimated values of the physical constants on which it is based. Predicted values of qb_{lee} correlates with the measured flume data values of bedform height multiplied by bedform migration rate with $R^2 = 0.79$. The model was validated by means of independent measurements based on a flume study as well as from a small natural river.

1 INTRODUCTION

During the 20th century, a number of sediment transport models designed to describe bed load and/or total load have been formulated e.g. Meyer-Peter & Müller (1948), Bagnold (1956), Bagnold (1966), Einstein (1950), Yalin (1963), Chang et al., (1967), to mention those evaluated by Yalin (1977) in his comprehensive overview. Some of the many other transport models which can be mentioned in this context are: Engelund & Hansen (1967), Ackers-White (1973), Engelund & Fredsøe (1976) and van Rijn (1984). Many others could be added with equal right. The vast majority of these models have been calibrated by means of the most comprehensive dataset ever published on sediment transport in flumes (Guy et al., 1966). So, as the one presented in this paper is also calibrated by means of this data set, it seems obvious to ask why the scientific community should be bothered by yet another Guy et al. (1966) calibrated sediment transport model. The primary reason for this is that we actually need a model to describe bedform migration as such, i.e. a model which liberates us from the need of using empirical bedload models referring to more or less blurred definitions of the bedload component (expressed as weight per unit time and width) when describing bedform migration. Also, the bedform migration data set in Guy et al. (1966) - to the best of our knowledge - has not previously been used to calibrate sediment transport models, which makes the present contribution a new approach, even if it is referring to an aged dataset.

In recent years, we have become increasingly better in measuring bedforms and their behaviour in nature (e.g. Kostaschuk & Villard, 1996; Kostaschuk, 2000; Bartholdy et al., 2002; Bartholomä, 2004; Bartholdy et al., 2005; Ernstsen et al., 2005, 2006, 2007; Duffy et al., 2005; Parsons et al. , 2005), and to translate results of detailed flume studies into relevant information for natural scale conditions (e.g. McLean et al., 1994, Bennett & Best, 1995, McLean et al. 1996, Best & Kostaschuk, 2002). Available bed load models, however, do not explicitly describe bedform migration. This has forced researchers with the need of evaluating bedform migration by means of a model, to use one of the traditional bedload models and subsequently recalculate weight to volume of loosely packed migrating bedforms (e.g. McLean et al., 1996). Bedload is per definition regarded as that part of the total load which is "carried by the bed" in contrast to suspended load which is "carried" by the turbulent flow (e.g. Yalin, 1977).

The borderline between the two, however, is difficult to define as there is a gradual transition from "hopping" bedload particles to "swimming" suspended grains. Sand will over time, be transported as both bedload and suspended load (intermitted suspension), not least when bedforms are present. For this reason, and as a result of a number of technical problems, it is actually not possible to measure the exact bedload share from a total load consisting of both transport modes. In the dataset of Guy et al. (1966) bedload is defined as the difference between the measured total load, obtained by means of width-depth-integrated water samples at the end of the flume, and the suspended load measured on the basis of depth integrated water samples in the flume. The data set also contains a detailed description of bedform migration. Focusing on the form migrating part of the total load gives, besides the obvious advantage of dealing with a well defined parameter, the benefit (for bedform studies) of a direct description of bedform behaviour.

A large part of literature dealing with sediment transport is published by hydraulic engineers in what sometimes appears less intelligible to sedimentologists who have a different focus. Furthermore, the critical conditions for initiation of motion are often related to a fixed (for sand-sized sediment often too high) critical Shields' parameter. The primary purpose of this paper is to present a simple, fully explained bedform migration model for dunes which includes a specification of the critical conditions for the initiation of motion. Another important task for the development of such a model is to extract that part of the total bed shear stress, which contributes to bedform migration. This is done by means of the procedure suggested in Bartholdy et al. (in prep. A). The present paper is an abbreviated pre-congress version of (Bartholdy et al., in prep B) planned to be published in the proceedings of MARID 2008.

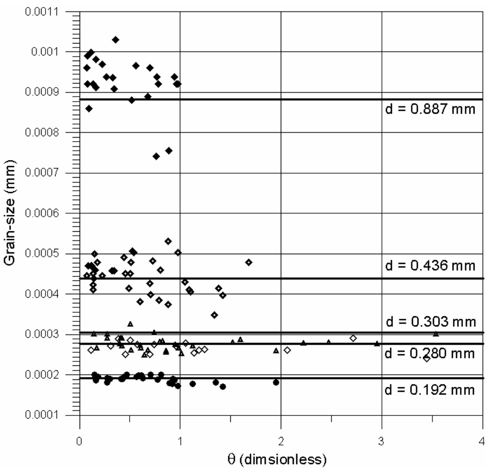

Figure 1. Measured grain size of bed material against the Shields' Parameter for runs from Guy et al. (1966) (wide flume with information about bedform dimensions).The horizontal bold lines represent the respective mean grain-sizes of the four sediment types. Only d = 0.303 mm (triangles) and d = 0.280 mm (diamonds) are difficult to differentiate.

The model quantifies the sediment load which is carried over the bedform top and deposited on the slip face to form a positive contribution to what can be measured as bedform migration. It is simple, built on a rational description of simplified sediment mechanics, and its calibration constant can be explained in accordance with estimated values of the physical constants on which it is based. The model is thought of as a modern analogue to previously published weight-based models. Input to the model are mean grain-size, bedform height, a measure of the current velocity (point or depth averaged), level of measured velocity or mean depth, and water properties (temperature and salinity). Output from the model is the bed form migration in form of the natural packed volume of sediment per unit time and width transported over the bedform top to be deposited in the trough region. This is directly translatable to the bedform migration rate by division with the bedform height.

2 METHODS

2.1 *Flume data*

The flume data by means of which the proposed model is calibrated consists of the data set published by Guy et al. (1966), often referred to as the "Fort Collins Data". It was chosen to use only data sets from the widest flume (8 feet or 2.43 m wide), in order to avoid effects resulting from too narrow boundaries. Also, only runs with clean water are used, avoiding those where clay has been added to the flume water. This leaves data from the following tables in the report: Table 2 (p. I62-I63), Table 3(p. I64-I65), Table 4(p. I64-I65), Table 5(p. I66-I67) and Table 6(p. I68-I69) of which 138 data sets are reported with information on bed form dimensions. Of these 76 are classified as dunes or transition between dunes and upper plane bed, and 71 have additional information about bed form migration 46 of which are classified as dunes. When using these data sets in the past, little attention has been given to the reported grain-size information. Considering the importance of these results for a large number of fundamental studies, it seems odd that few, if any, have shown interest in this important parameter. As a rule, the reported median diameter (d_{50}) of "several 0.1-foot-diameter cores, about 0.6 foot in length, taken at random from the bed of the flume" (Guy et al. 1966, p. I8), has been used as a basis for the single data sets. Plotting these d_{50} values against the Shields' Parameter, gives no significant trend, and the d_{50} seems randomly distributed about a certain mode for each bed sediment type (Fig. 1). On the basis of this, the variation in d_{50} is interpreted as reflecting poor sample representativity rather than genuine differences.

Thus, it does not seem justified to use these values as a better measure of the actual bed material grain-size of the single runs, than the average grain-size of all runs related to a certain bed material type. Furthermore, not all runs have a reported d_{50}. As on top of this, the average bed-material type has been treated in detail in the report (Guy et al., 1966; Fig. 3 p. I4), this allows the use of mean grain size in stead of d_{50} of the bed material (Table 1). It is important to notice, that these grain-size distributions are based on fall diameters (the diameter of a quartz sphere with the same fall velocity). In Table 1. a translation of the reported fall diameters to sieve diameters has been carried out following Yin et all. (1999).

Table 1. Statistical parameters (Folk & Ward, 1957) of mean bed-material (from Guy et al. (1966). d is mean grain-size (mm), Sd is the standard deviation (phi), Sk is the skewness (dimensionless) and Kg is the kurtosis (dimensionless). Note that all grain-size parameters except d(sieve) are based on fall diameters.

Parra-meter	Type 0.19 mm	Type 0.27 mm	Type 0.28 mm	Type 0.45 mm	Type 0.93 mm
d(fall)	0.192	0.280	0.303	0.436	0.887
Sd(phi)	0.386	0.699	0.851	0.678	0.663
Sk	-0.006	-0.222	-0.329	0.068	0.196
Kg	0.998	1.160	1.292	1.066	1.024
d(sieve)	0.180	0.284	0.314	0.490	1.167

This translation is almost identical to the one suggested by Flemming and Ziegler (1995), and thus seems to be generally valid for "natural worn particles".

2.2. *Critical conditions for the initiation of motion*

As an appropriate description of the critical conditions for initiation of sediment transport is crucial for any sediment transport model, and is usually expressed in terms of the critical Shields' parameter:

$$\theta_{cr} = \tau_{cr} / [(\rho_s - \rho)\, g\, d] = uf_{cr}^{\;2} / [(s-1)\, g\, d] \qquad (1)$$

plotted against the grain the Reynolds number:

$$R_* = uf \cdot d / \nu \qquad (2)$$

where τ_{cr} = the critical bed shear stress; ρs-ρ = the submerged density of sand; g = the acceleration due to gravity; d = the mean grain size; uf_{cr} = is the critical friction velocity; s-1 = the dimensionless submerged density of sand; and ν = the kinematic viscosity.

The curve for initiation of transport in uniform material was first suggested by Shields (1936). Thereafter numerous other results have been published culminating in the thorough and often cited work by Yalin & Karahan (1979). In Fig. 2 this curve has been plotted in a diagram also showing isolines for friction velocity (*uf*) and mean grain-size (*d*), under the assumption of clean water and a constant temperature of 10°C. This grid of *uf* and *d* will move sideways with temperature variations (to the left under cooling). Many sediment transport models concentrate on the fact that θ_{cr} becomes constant for large values of R_*, and hence use a constant value of θ_{cr} (e.g. 0.047 and 0.05 in the models of Meyer Peter & Müller, 1948 and Engelund & Fredsøe, 1976, respectively). This however can be relatively far from the correct value when dealing with sand-sized material, as the curve minimum (θ_{cr} = 0.032) falls in the medium to coarse sand fraction (Fig. 2).

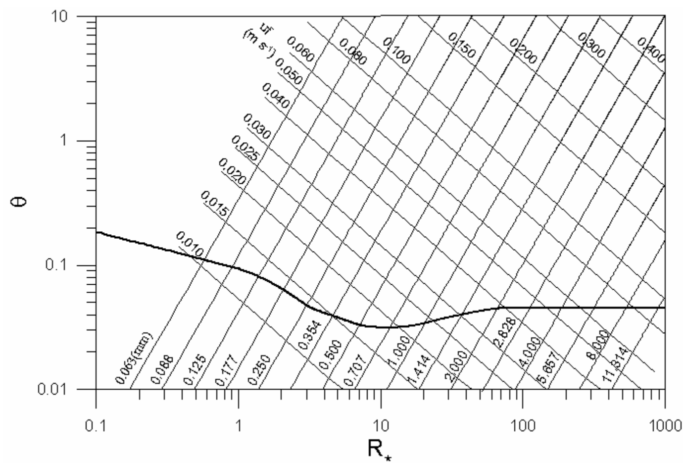

Figure 2. Critical conditions (bold line) for the initiation of sediment transport in uniform grain-size based on the results of Yalin & Karahan (1979). The algorithms related to the digitized curve appear from Table 2. Isolines for friction velocity (*uf*) and mean grain-size in mm is added under the assumption of clean water and a constant temperature of 10°C.

The curve is relatively accurately described (R^2>0.99, corresponding to a digitized adaptation based on a magnified version of the original Fig. 1 on p. 1434 in Yalin & Karahan, 1979) by means of the algorithms shown in Table 2.

Table 2. Algorithms for relations between grain Reynolds number and the critical Shields' parameter as shown in Fig. 2.

Grain Reynolds number, R_*	Algorithm; θ_{cr} =
< 1.0	$0.093\, R_*^{-0.3}$
1.0 - 3.7	$0.13271 - 0.04530\, R_* + 0.00566\, R_*^2$
3.7 - 9.0	$0.06261 - 0.00690\, R_* + 0.00038\, R_*^2$
9.0 - 30	$0.03858 - 0.00145\, R_* + 0.000087\, R_*^2 - 0.0000013\, R_*^3$
30 - 70	$0.02621 + 0.00049 R_* - 0.0000032\, R_*^2$
> 70	0.045

Such an unambiguous description of the curve is needed if - when dealing with sand sized material - less accurate readings of the empirical curve are to be avoided. The algorithm for R_*<1 was suggested by Yalin and Karahan (1979) themselves. The values of θ_{cr} and uf_{cr} are in the following based on an iterative procedure defining θ_{cr} as a function of R_* with temperature (*t*) and mean grain size (*d*) as independent parameters, and controlled by the equations stated in Table 2. The kinematic viscosity for clean fresh water was found by dividing the viscosity (μ) with the density (ρ) - both adapted from values listed in Weast (1972). The following two polynomial regressions, both correlating with R^2 =1.00, describe these parameters as a function of temperature:

$$\mu = 0.001769778702 - 5.232388614\cdot10^{-5}t + 6.787908282\cdot10^{-7}\ t^2 \quad (3)$$

$$\rho = 999.9121701+0.03591225099\ t - 0.006491429872\ t^2 \quad (4)$$

2.3. *The effective bed shear stress*

The form-corrected bed shear stress due to skin friction is calculated as the bed shear stress acting on top of the dunes. It is based on the method suggested by Bartholdy et al. (in prep. A) who rendered the existence of a virtual horizontal boundary layer of D' = 2H above the mean bed level, with H being the bedform height.

According to this method, water can be regarded as entering the boundary layer at mean bed level midway between the trough and top with a mean velocity, V', which - according to the logarithmic velocity profile (e.g. Yalin 1977) - can be expressed as,

$$V' = (6 + 2.5\ln(2H/k_s))uf \quad (5)$$

where uf = the overall friction velocity; and k_s = the overall hydraulic roughness :

$$k_s = k_{skin} + k_{form} \quad (6)$$

the skin roughness is described according to Engelund & Hansen (1967) as:

$$k_{skin} = 2.5\ d \quad (7)$$

and the form roughness is described by Bartholdy et al. (in prep. A) as:

$$k_{form} = 0.57\ H \quad (8)$$

The flow accelerates towards the bedform top in the narrowing layer where, for continuity reasons, it becomes:

$$V'_{top} = V'(2/1.5) = 4/3V' \quad (9)$$

as the layer thickness here is 1.5H.

From this, the local friction velocity at the bedform top is calculated on the basis of the logarithmic velocity profile with k_{skin} as roughness parameter:

$$uf_{top} = V'_{top}/(B - 2.5 + 2.5\ln(1.5H(k_{skin})) \quad (10)$$

As most flow conditions close to the bed over the top of bedforms belong to the transitional regime between rough and smooth flow, the usual constant, 6, added to the logarithmic term in the theoretical velocity equation, is in (10) replaced by B-2.5. The figure 2.5 is a constant stemming from integrating the logarithmic velocity distribution, and for $1.6<R<70$ ($R = uf_{top}k_{skin}/\nu$, where ν is the kinematic viscosity) B can be expressed in form of a 3rd degree polynomial approximation:

$$B = 4.52 + 11.19\log(R) - 7.83(\log(R))^2 + 1.59(\log(R))^3$$

for R>70, B = 8.5.

The local form-corrected dimensionless bed shear stress due to skin friction acting on top of the dune then becomes:

$$\theta_{top} = uf_{top}^2/((s-1)\ g\ d) \quad (11)$$

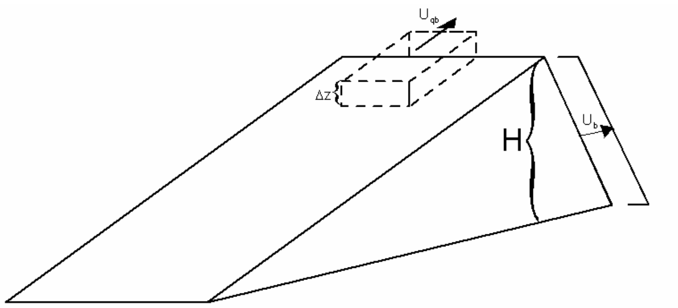

Figure 3. Illustration of the sediment transport model concept.

3 BED FORM MIGRATION MODEL

3.1. *Formulation of the basic concept*

If, over the top of a bedform (Fig. 3) the material contributing to bedform migration can be regarded as transported in a Δz thick layer with the bulk dry density V_{01}, the volume of solid material per unit area of this layer is: $\Delta zV_{01}/\rho_s$. The frictional resistance stress corresponding to a unit area of this layers passage over the bed is therefore

$$F_f = (\Delta zV_{01}/\rho_s)(\rho_s - \rho)g\alpha \quad (12)$$

where ρ = the density of water; ρs = the density of the sediment; g = the acceleration due to gravity; and α = the frictional coefficient between the layer and the bed. With a layer velocity of U_{qb}, the corresponding streampower is

$$E_f = (\Delta zV_{01}/\rho_s)(\rho_s - \rho)g\alpha U_{qb}$$

$$= qb_{top}(V_{01}(\rho_s - \rho)/\rho_s)g\alpha \qquad (13)$$

where $qb_{top} = \Delta z\, U_{qb}$ is the transported volume of this layer passing the bedform top per unit time and width.

As this transport is driven by the bed shear stress acting on top of the bedform, τ_{top}, free of form drag, the corresponding power which keeps the layer moving is

$$E_\tau = \tau_{top} U_{qb} \qquad (14)$$

Assuming a time-averaged steady state movement, the average values of E_f and E_τ must balance each other out:

$$qb_{top}(V_{01}(\rho s - \rho)/\rho s)g\alpha = \tau_{top} U_{bq} \qquad (15)$$

=>

$$qb_{top} = \tau_{top} U_{bq}(V_{01}(\rho s - \rho)/\rho s)^{-1} g^{-1}\alpha^{-1} \qquad (16)$$

The sediment transport is zero if the friction velocity is equal to or below the critical friction velocity. Under the assumption, that the velocity of the layer is proportional to the surplus friction velocity corresponding to the bed shear stress acting on top of the bedform, the velocity of the moving layer for values of friction velocities larger than the critical one, uf_c is

$$U_{qb} = b\,(uf_{top} - uf_c) \qquad (17)$$

where b is a dimensionless constant.

Inserting the right-hand side of Equation 17 as U_{qb} in Equation 16 gives

$$qb_{top} = \frac{b\rho s}{V_{01}(\rho s - \rho)} \frac{\tau_{top}}{g}(uf_{top} - uf_c) \qquad (18)$$

The dry weight of the transported bedload material (per unit time and width) over the bedform top is $qb_{top} \cdot V_{01}$. As this, in accordance with the aim of the model, corresponds to the material contributing to the bedform migration by deposition on the lee slope of the bedform, we can write

$$qb_{top} V_{01} = qb_{lee} V_{02} = H\, U_b\, V_{02}$$

=>

$$qb_{top} = qb_{lee} V_{02}/V_{01} = H\, U_b V_{02}/V_{01} \qquad (19)$$

where V_{02} = the dry bulk dry density of the material building up the bedform; Ub is the bedform migration rate; and qb_{lee} is the volume of natural packed sediment per unit time and width participating in the bedform migration (equal to $H\,Ub$).

If Equation 19 is used to replace qb_{top} in Equation 18, and the right-hand side of Equation 18 is multiplied by the grain size, d, in both numerator and denominator, the expression reads:

$$HU_b \frac{V_{02}}{V_{01}} = \frac{bd\rho s}{V_{01}\alpha} \frac{\tau_{top}}{(\rho s - \rho)gd}(uf_{top} - uf_c)$$

=>

$$HU_b = \frac{bV_{01}\rho s}{V_{02}V_{01}\alpha} d \frac{\tau_{top}}{(\rho s - \rho)gd}(uf_{top} - uf_c)$$

=>

$$qb_{lee} = \beta d\,\theta_{top}(uf_{top} - uf_c) \qquad (20)$$

where β is a dimensionless constant,

$$\beta = \frac{b\rho s}{V_{02}\alpha}$$

For high values of uf_{top} ($uf_{top} >> uf_c$) the bed load transport over the bedform crest (qb_{lee}) expressed in this way is directly proportional to uf_{top}^3 or $\theta_{top}^{3/2}$. Thus, Equation 20 corresponds to the well known empirical bed load formula of Meyer Peter & Müller (1948), in which the bedload transport is proportional to $((\theta' - \theta_c)\,d)^{3/2}$. However, the two models are not identical, and the model presented here provides a simple and sound physical explanation for the proportionality factor, β, as well as linking the dynamical parameters directly to bedform migration.

3.2 *Calibration of the bedform migration model*

The model, as expressed in Equation 20, has been calibrated on the basis of 46 data sets from Guy et al. (1966) with reported bedform migration rates. The calculations were based on measured mean velocity over depth, depth, bedform height, grain size and bedform migration rate. The former two - with grain size and bedform height - were transformed into uf, by means of the logarithmic velocity equation with ks found from Equation 6, whereas uf_{top} was found as described in Section 2.2, and ufc by iteration using the algorithms listed in Table 2. As shown in Fig. 4, β is in this way calibrated to a value of 27.0 which gives Equation 20 the following final appearance:

$$qb_{lee} = 27d\,\theta_{top}(uf_{top} - uf_c) \qquad (20')$$

The calibrated equation correlates with the measured values of HU_b with $R^2 = 0.79$. If in accordance with numerous studies (e.g. Bartholdy et al., 1991; Carling, 2000; Fredsøe & Daigaard, 1992) the value of V_{02} is regarded as being approximately 1700 kg

m^{-3} and α (from the angle of repose) is put to tan 27° = 0.51 the dimensionless constant, b, linking (uf_{top}-uf_c) to the bedload transport velocity, U_{qb}, becomes 8.8 as a consequence of a calibration constant of 27. Following the results of Luque (1975), Engelund & Fredsøe (1976) suggested U_{qb} to be equal to 9.3(uf'-0.7ufc), uf' being their effective (grain related) friction velocity. Using the here suggested "calibration result" of 8.8(uf_{top}-ufc), give values of U_{qb} of less than 20% below this for θ_{top}>0.4.

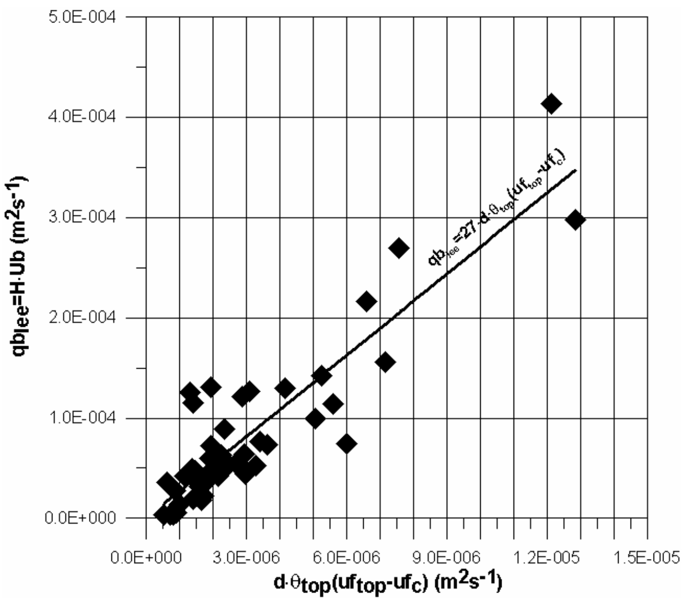

Figure 4. Relation between bedload measured as bedform migration and the expression of Eq. 20 without the constant β for all runs from Guy et al. (1966) with the wide (8 feet) flume and reported dune migration. From this plot β is calibrated to 27.0 which correlates with R^2 = 0.79.

Thus, if, β in Equation 20 was based solely on independent (from literature) estimates of V_{02}, α and b; an un-calibrated version of the equation would give what can be considered to be as at least comparable results. It should here be kept in mind here that the two different methods of estimating U_{qb} are based on different ways of estimating both the effective and the critical shear stress.

The fact that is possible in this way to explain the calibration constant in the proposed bedform migration model based on known independent estimates of well defined physical parameters, is regarded as a relatively strong argument for the applied approach.

4 EVALUATION

Not many datasets on steady-state simple subaqueous dune-migration, as well as all the other parameters necessary to run the model, are available from the literature. Papers dealing with bedforms in nature, often concentrate on the largest measurable bedforms without reference to possible superimposed smaller dunes. Even if the large ones are known to be compound, information about the smaller bedforms on their back is generally missing. In order to evaluate the proposed model on an independent data set at this stage, we use data from a

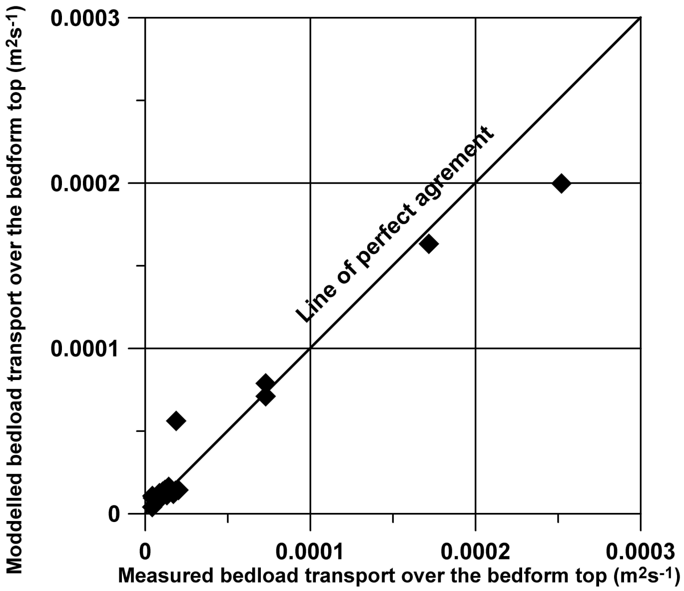

Figure 5. Measured versus modelled bedload over the bedform top. Data from Iseya (1984) and Erntsen (2002).

flume study by Iseya (1984), and a yet unpublished dataset from the river Gels Å in western Denmark (Ernstsen, 2002) based on one of the authors MSc-thesis. We are currently interested in collecting additional suitable datasets, especially from the field, and hope to improve our collection through contacts at the MARID meeting in Leeds.

The experiments of Iseya (1984) were conducted in a 4 m wide and 160 m long recirculating flume with sand having a median fall diameter of 0.57 10^{-3} m. The dune dimensions were measured by means of sonic sounder depth profiles along both sides of the central 1/3 of the flume. The surface velocity over the dunes was measured by means of floats. The speed of dune migration was obtained from successive measurements of the position of dune brinkpoints through a Plexiglas wall (20 m long) of the flume. The steady state dune migration was measured in 5 runs with reported friction velocities between 0.062 m s^{-1} and 0.096 m s^{-1}.

Ernstsen (2002) reported measurements from the central part of a small river about 12 m wide with water depths varying between 0.72 m and 0.83 m and mean current velocities between 0.43 m s^{-1} and 0.58 m s^{-1}. The bed material mean fall diameter of 0.42 10^{-3}m was translated from the sieve diameter by means of the method presented in Yin et al. (1999). Bedform migration was calculated from bed profiles recorded successively by a single beam echo sounder moving on a 4.8 m long rail mounted on anchored pontoons. The bed profiles were measured at half an hour intervals (in principle 3 times a day) over about a month in the spring of 2001. The vertical precision of the measurements was +/-0.5 cm. An Aanderaa RCM9 acoustic current meter mounted close to 0.4·D above the bed kept track with water temperature and mean current velocity (at 5 min. intervals). The velocity data were adjusted to give the mean velocity by means of the logarithmic velocity distribution. Only observations with dunes (defined as L>0.60 m) were used.

Measured and modelled transport rates from these two datasets are shown in Fig. 5. The bedload transported over the top of the bedforms correlates with R^2 = 0.96. The material is too thin in order to form the basis of a proper statistical analysis. On the other hand, the relatively good fit supports the proposed model. Note that the modelled bedload transport here is that passing over the bedform top. In order to translate this into the total bedload transport of natural packed sediment per unit time and width, it should be multiplied with the bedform shape factor which for triangles is equal to 0.5 and for natural bedforms somewhat larger up to about 0.6 (e.g. van Rijn, 1993).

4 ACKNOWLEDGEMENT

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