Modelling sand wave migration and height, comparing model results and data

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Abstract

Sand waves form a prominent regular pattern in the offshore seabed of sandy shallow seas. The positions of sand-wave crests and troughs slowly change in time. Sand waves are usually assumed to migrate in the direction of the residual current. This paper considers the applications of two models assuming that sand waves evolve as free instabilities of the system on two different cases. Both morphological models consider two-dimensional vertical (2DV) flow and include the interaction between the vertically varying water motion and an erodible bed in a shallow sea and contain the same basic equations. Firstly, a linear stability analysis is performed to investigate a field of migrating sand waves of the Dutch coast. The order of magnitudes found of the time and spatial scales coincide with the observations. Secondly, to investigate the value of the obtained results on sand wave migration for sand waves having finite heights, we used a numerical simulation model to investigate the properties of these bed forms during their evolution, for a unidirectional steady current situation in the Spanish Gulf of Cadiz. The migration rate of the sand waves appears to decrease only slightly during their evolution. This indicates that the linear stability analysis, used in the North Sea case study, provides a good estimate of the migration rate. Furthermore, the sand waves become asymmetrical during their evolution and saturate at 10-30% of the average water depth on a timescale of decades.

1. Introduction

Offshore sand waves have typical wavelengths of several hundreds of metres and can be found in shallow seas. Their crests are oriented more or less perpendicularly to the principal direction of the current (Hulscher, 1996). The heights of sand waves can grow up to 30% of the average water depth. Therefore, the relative sand wave height can be considered as significant. Observations indicate that these sand waves are dynamic (Lanckneus and De Moor, 1991; Allen, 1980) and can migrate with speeds of up to several metres per year.

Insight into the behaviour of these sand waves is crucial to enable cost-effective management practices. Due to the large height of sand waves compared to the water depth in combination with the timescale of years on which they are assumed to be active, they play an important role with respect to for example pipelines and cables in coastal seas (Németh et al., 2003).

Knaapen and Hulscher (2002) developed an evolution model based on data assimilation, and investigated data sets of a field of sand waves near Japan. This analysis showed that when a sand wave is dredged, it is able to recover on a timescale of about eight years. Morelissen et al. (2002) extended this model by allowing sand waves to migrate using a modified Landau equation. Despite the success of this empirical method, it does not include the full knowledge of sand wave physics, as the physical basis of the nonlinear processes was lacking at that moment. Therefore, it cannot facilitate the investigation of the mechanisms leading to non-linear sand wave behaviour.

Based on Huthnance (1982), Hulscher (1996), Gerkema (2000) and Komarova and Hulscher (2000)), Németh et al. (2002) developed a model describing the formation and migration of infinitely small sand waves based on a stability analysis taking periodic water motion (M2) in combination with a steady part

(M0) into account. The model gives insight into the initial evolution and migration of sand waves assuming they are free instabilities of the seabed water system. Besio et al. (2003) extended this research by including an M4 tidal constituent. Furthermore, Besio and Blondeaux (2003) incorporated the effect of suspended sediment transport in their linear analysis.

Komarova and Newell (2000) investigated the model by Hulscher (1996) combined with the time-dependent viscosity parameterisation from Komarova and Hulscher (2000) using a weakly non-linear analysis. This analysis led to coupled spatial variations of sand waves and the average bed level, of which the latter shows similarities with tidal sandbanks. However, sand wave migration cannot be investigated with this analysis.

Johns et al. (1990) and Stansby (1998) discussed unidirectional steady water movement over dune-like features with steep slopes. Due to the steep slopes, the focus is on flow separation, which we do not expect for offshore sand waves with a smaller steepness than dunes found in rivers. Fredsøe and Deigaard (1992) discuss the behaviour of sand waves based on a model describing fully-developed sand dunes in rivers. Sand waves under the influence of oscillatory water movement are schematised as bed forms formed under the influence of a unidirectional steady current, with a modification for the reversing current. Therefore, periodic water motion is not taken directly into account. Furthermore, the model does not describe bed form evolution. Richards and Taylor (1980) discussed flow and sediment transport characteristics for more sandwave-like bed forms in a unidirectional steady flow, with milder slopes than river dunes. In addition, they discuss the response of the seabed for various shapes of the sand wave. Idier (2003) investigated sinusoidal sand waves for various amplitudes with a numerical model for unidirectional steady flow conditions.

In Németh (2003), a numerical model has been developed and validated mathematically with the help of a linear stability analysis (Németh et al., 2002). This model enables the description of the entire evolutionary process of sand waves.

Within this paper we will investigate whether we can estimate characteristics of sand waves using idealized models, by comparing the results with data from a sand wave field in the North Sea and in the Gulf of Cadiz in Spain. For the North Sea case we will investigate if we can estimate the wavelength and migration rate of the sand waves using a stability analysis (Németh et al., 2002), formally only valid for small amplitude sand waves. Next, we will assess the value of these results, focusing on sand wave migration, by allowing sand waves to become finite using a simulation model (Németh, 2003), based on sand waves found in the Gulf of Cadiz in Spain.

First, small amplitude sand waves are investigated for a North Sea case in section 2. Next sand waves of finite extent are investigated for the Gulf of Cadiz case (section 3). In the final section (4), we present the conclusions.

2. North Sea – Small amplitude Sand waves

2.1 Bathymetric surveys along a North Sea pipeline

The bathymetric data have been digitized from pipeline alignment sheets. We used data from one section of about 9 km of a pipeline in the southern Bight of the North Sea. Hereby, the crests of the sand waves lie almost perpendicular to the pipeline and the principal direction of the current. Five surveys were available and carried out in 1995, 1996, 1998, 1999 and 2000, giving a total time span of 6 years.

In general the position of a pipeline is very stable. Therefore, it provides a reliable reference position for the bathymetric measurements on charts. This reference position is important to provide accurate estimates of sand wave migration. The position of the pipeline itself is given by only a couple of measurements. The total error of the horizontal positioning is expected to be less then +/- 10 m. The total error of the vertical position of the seabed lies in the order of 0.2 m. Both the horizontal and vertical error are equally divided over measurement error and digitisation error (For more information see Morelissen et al. (2003)).

2.2 Data pre-processing

To compare the measured data with results obtained with the sand wave model, the mean bed profile has been derived from the data set using a low-pass filter based on a Hanning window. Subsequently, this mean bottom profile was subtracted from the original data to isolate the sand wave profile (see Fig. 1).

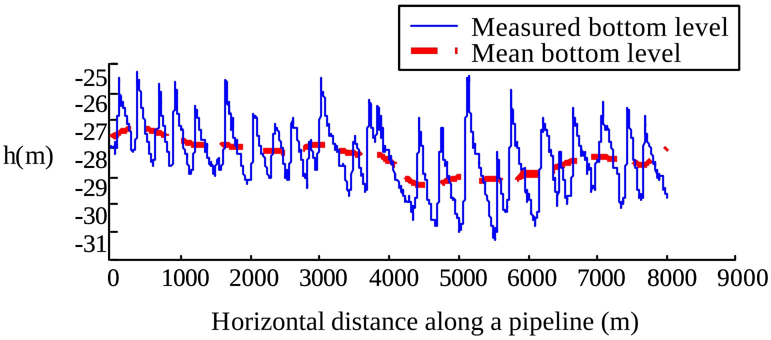

Fig. 1 – The original data (thin solid line) and the mean bottom profile (thick dotted line) found using the low pass filter. The bed level on the vertical axis and the position along the pipeline on the horizontal axis are in metres. The short bed level undulations are mega-ripples superimposed on the sand waves. These mega-ripples fall outside the scope of this paper (After Morelissen et al. (2003)).

Looking at Fig. 1, the wavelengths of the sand waves are in the order of 400 m. However, in reality these wavelengths are slightly smaller since the crests of the sand waves are not exactly oriented perpendicular with respect to the pipeline, but at a small angle. The sand waves have an average and maximum height of about 3 m (10% of the average water depth) and 6 m (20% of the average water depth), respectively. Furthermore, they are asymmetrical oriented to the North, coinciding with the residual current present in the Southern Bight of the North Sea (Dronkers et al., 1990) and with the general direction of movement of sand waves according to Houbolt (1968). This coincides further with the found migration of the sand waves in the northerly direction (left side of Fig. 1). The migration rate of the sand waves was assessed by a comparison of successive data sets and varies over the domain. Between 0 & 1000 m and 7000 & 8000 m the migration rate is found to be about 10 m yr^{-1}. Between 2000 and 3000 m, where the sand waves are less high compared to the remaining part of the data set, the migration rate found is about 20 m yr^{-1}. These rates are well within the range of values reported in the literature (See Lanckneus and De Moor (1991) and Allen (1980)).

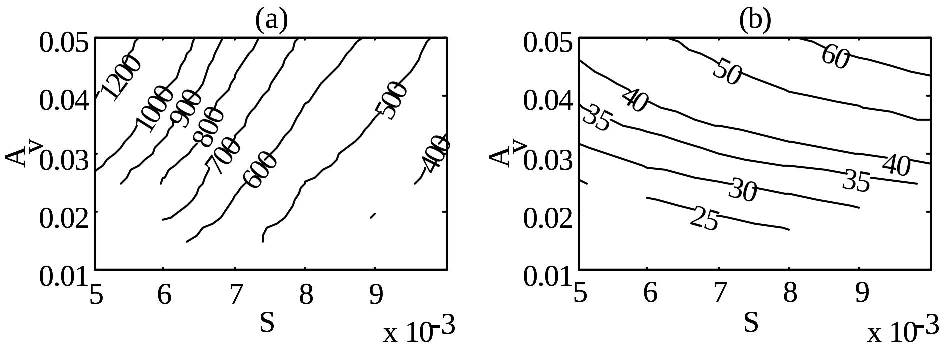

Fig. 2 – Properties of the fastest growing mode as a function of the resistance parameter *S* for different values of the eddy viscosity A_v: (a) wavelength (m) and (b) migration rate (m yr^{-1}).

2.3 Application and results stability analysis

We investigated with the linear stability analysis (Németh et al., 2002) periodic water motion with a depth-averaged amplitude of 0.9 m s^{-1} together with a steady current of 0.1 m s^{-1}, based on a pressure gradient, as an estimate of the flow conditions present at the location of the data set. Furthermore, the sediment diameter is about 2.4 10^{-4} m (Baptist et al., 2001). The average water depth is estimated to be about 28 m. The other parameter values used are equal to the ones used in Németh et al. (2002).

The wavelength and coinciding migration rate of the fastest growing mode (FGM) as a function of the resistance parameter S for various values of the eddy viscosity A_v can be found in Fig. 2. A typical North Sea location has a value of the resistance parameter S of about 0.006-0.008 m s^{-1} and an eddy viscosity A_v of about 0.01 m s^{-1}. This gives us according to the linear stability analysis a wavelength in the order of 400 m and a coinciding migration rate of about 20 m yr^{-1}.

3. Gulf of Cadiz in Spain – Sand waves of finite extent

In this section we will investigate the evolution of sand waves with a fully coupled model (Németh, 2003). This means that the seabed is allowed to change during the long-term morphological calculations. Hereby, we will use periodic boundary conditions. Using this approach, we choose the length of the domain to be equal to the initially preferred wavelength based on linear theory. Therefore, this wavelength is fixed during the morphological calculations.

In the previous section we were looking at sinusoidal sand waves, whereby it is common to refer to the magnitude in the vertical using the term amplitude. However, in this section the bed forms are not sinusoidal anymore. Therefore, for non-sinusoidal bed forms we will use the height, defined as the distance from the trough to the top.

3.1 Sand waves in the Gulf of Cadiz

In Spain in the Gulf of Cadiz sand wave like bed forms in a predominantly unidirectional steady flow are found in average water depths of 20 metres on a continental shelf of about 30 km wide. Due to the nature of the tidal motion in combination with the shape of the coastal environment, the ebb and flood parts of the tidal motion pass over different areas in the Gulf. This distinguishes the area from the North Sea where periodic water motion is dominant, which is sometimes modified by a steady component. In the Gulf of Cadiz we find offshore sand waves existing in unidirectional more or less steady flows.

The wavelengths and heights of the sand waves are typically 150-300 m and 2-4 m, respectively. Both symmetrical and asymmetrical sand waves are found. In Table 1 the characteristics can be found based on one measurement of the bathymetry. Therefore, no information concerning their migration rates is available. Rommel (2002) investigated these bed forms using the model by Németh et al. (2002). Here, we will investigate sand wave migration, evolution and saturation in the gulf of Cadiz.

Parameters	**A**	**B**	**C**	**Dimension**
Mean grain size diameter	4.15	5.37	0.58	10^{-4} m
Steady current velocity	1.5	0.8	0.2	m s^{-1}
Average water depth	19	21	22	m
Range sand wave length	100/850	50/4000	50/350	m
Mean sand wave length	275	190	145	m
Range sand wave height	1/11	1/6	1/5	m
Mean sand wave height	3.75	2	1.9	m
Asymmetry ratio	1/5.6	1/7.5	1/6	-

Table 1: Characteristics for the different areas under investigation of the Gulf of Cadiz (See also (Lobo et al., 1995) and (Nelson et al., 1999)).

In zone A the largest sand waves have average wavelengths of bout 275 m and average heights of 3.75 m (see Table 1). In zone B we find the most asymmetrical sand waves in the Gulf; the asymmetry is defined as the horizontal length of the lee side divided by the horizontal length of the stoss side. Between these two zones we can distinguish a transitional zone where sand waves are symmetrical. Zone *C* contains relatively smaller scaled sand waves, with wavelengths of about 145 m and heights with a maximum of 2 m, with rounded and eroded crests.

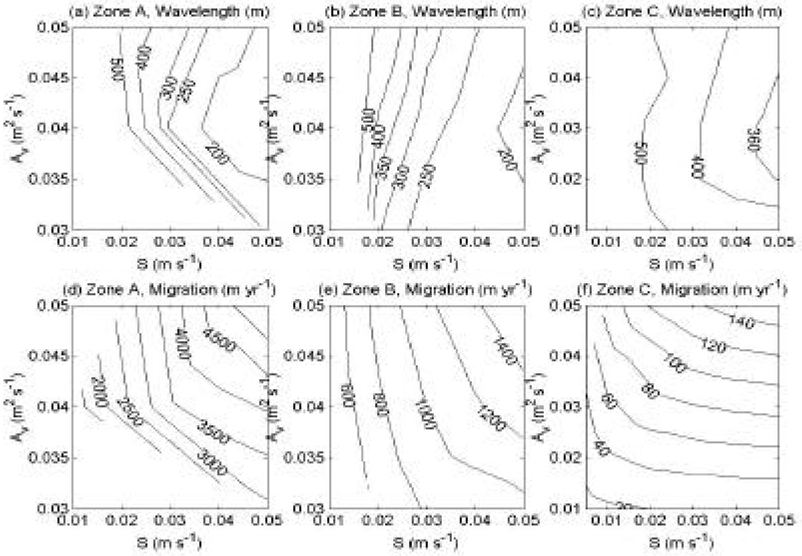

Fig. 3 – Fastest growing modes and coinciding migration rates for three zones in the Gulf of Cadiz as a function of the eddy viscosity A_v and resistance parameter S.

3.2 Linear analysis

Figs. 3 (a), (b) and (c) show the wavelengths of the fastest growing modes according to the linear stability analysis for zones *A*, *B* and *C*. This for a range of values of the resistance parameter *S* and the eddy viscosity A_v. These two variables are considered to be the most difficult to estimate. The rest of the values of the parameters can be found in Table 1. The calculated wavelengths are slightly longer than the actual wavelengths found. Since the order of magnitude is correct and the available data is limited we can conclude that the approach is successful, indicating the bed forms found in the Gulf of Cadiz can be free instabilities of the system.

3.3 Sand wave evolution

In Figs. 4 (a) and Fig. 5 (a) the development of the seabed in zone *C* as a function of time is presented. We started with the fastest growing mode determined with the linear stability analysis (Németh et al, 2002) for a steady current induced by a pressure gradient and a value of the resistance parameter S of 0.04 m s^{-1} and the eddy viscosity A_v of 0.02 m^2 s^{-1} (see also Table 1 and Fig. 3).

Using this model we can identify the evolution of the sand wave pattern. In Fig. 4 (a) the positions of the seabed in all the grid points are plotted as a function of time. The distortion apparent is not a numerical error or higher harmonic in the seabed topography. This is due to the narrowing grid density near the boundary of the computational domain where the sand waves pass through while migrating. The evolution of the height of the sand wave according to the current model is smooth (See also Morelissen et al. (2003).

The initial amplitude of the imposed perturbation is 1 mm. Initially, the bed form develops slowly in absolute sense, after which the amplitude increases faster, equivalent to the exponential solution of the linear stability analysis. Next, the growth rate diminishes and saturation is found. Based on the parameters used in this study, it takes about 30 years to develop from 10% to 90% of the saturation height of about 5 m. The final height yields 22% of the average water depth. The found height and wavelength of the sand waves lie within the range of the observations.

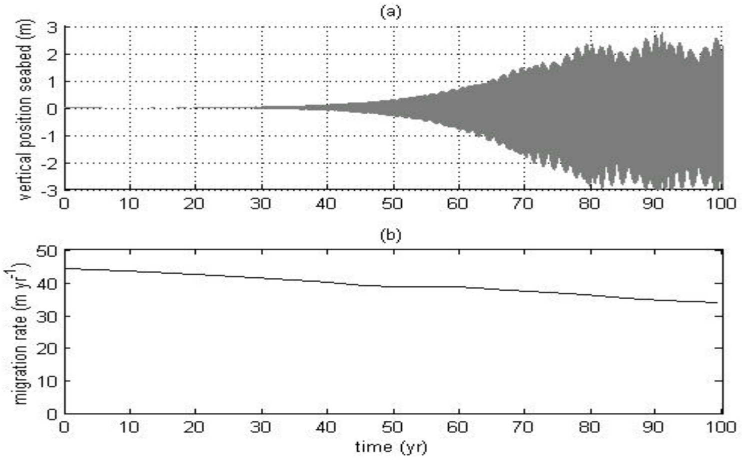

Fig. 4 – Evolution and migration of a sand wave in zone *C* of the Gulf of Cadiz. Fig. (a) shows the evolution of the sand wave and Fig. (b) the corresponding migration rate its the evolution.

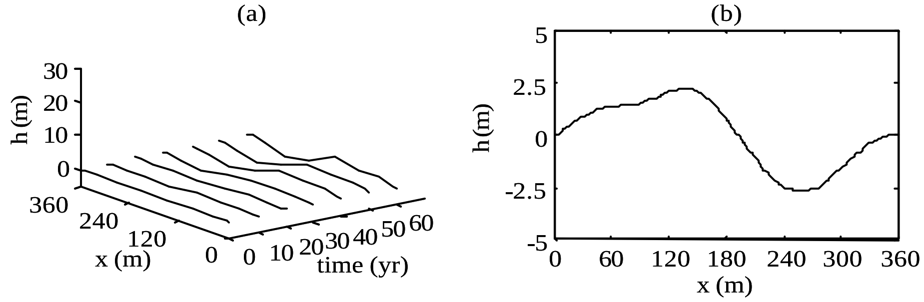

Fig. 5 – Cross section of a sand wave during its evolution in zone *C* of the Gulf of Cadiz. Fig. (a) shows the evolution in time of the seabed and Fig. (b) the cross-section of the saturated sand wave.

The slopes found are still small (the maximum is about 4%). Therefore, no flow separation occurs. This coincides with the observation that the bed forms in this zone *C* are more symmetrical than in the other zones *A* and *B*. The fully developed bed pattern is migrating with a migration rate of about 36 m yr^{-1}. The cross section shown in Fig. 5 (b) is the one after the sand wave has fully developed. However, the asymmetric shape is not yet in equilibrium, and changes further.

Similar calculations have been performed for zones *A* and *B* for different values of the resistance parameter *S* and the eddy viscosity A_v. However, for all cases the asymmetries of the bed forms become so large during their evolution, that we were not able to describe their entire evolution without flow separation. This is due to the larger current velocities/shear stresses. The shallow water approximation does not allow us to investigate the evolution further. The strong asymmetry the simulation model predicts coincides with the asymmetry of the sand waves found in zones *A* and *B*. Furthermore, this is analogous to the behaviour and physics of river dunes, where flow separation plays an important role.

3.4 Sand wave migration

The migration rate is defined as the velocity of the position of the lee side located at the average water depth. This definition is chosen since we are looking at a unidirectional steady current, inducing an asymmetric shape not equal to the sinusoidal sand waves investigated with the linear stability analysis. The migration rate (See Fig. 5) decreases slightly, from 44 m yr^{-1} for an amplitude of 1 mm to 36 m yr^{-1} for the fully-developed sand wave. This means an 18% difference between the result from the linear stability analysis and the fully-grown sand wave, which is not unusual for weakly non-linear phenomena, in which the deviation from the linear values is of the order of the non-linearity (here about 20%).

4. Conclusions and discussion

The results from the stability analysis coincide well with the observations made along a pipeline in the North Sea although the wavelengths are slightly too long. This deviation is reasonable, considering the simplifications in the model, the errors in the estimated values of the different parameters and the errors in the data set.

Using the numerical model, we are able to allow sand waves to evolve and become saturated. They reach heights of 10-30% of the average water depth for mild unidirectional flow conditions (for periodic water motion see Németh (2003)). It takes about 30 years to evolve from 10% to 90% of the saturation height. This coincides with values reported in the literature (See Knaapen and Hulscher (2002) and Idier (2003)). For moderate and highly energetic conditions flow separation occurs during the evolution of the sand wave. This is analogous to river dune dynamics. The shallow water approximation does not allow for flow separation. Therefore, the developed simulation model is in these cases not valid as can be expected.

Furthermore, the migration rate becomes slightly smaller during the evolution from an infinitely small to a fully-developed sand wave. This is as can be expected from a weakly non-linear problem where we expect the change in migration rate to be of the order of the relative sand wave height. This indicates that the linear stability analysis can give a good estimate of the migration rate, which can be advantageous since the calculation time required for this analysis is smaller than the time required using the simulation model.

This model approach forms a step in the direction of estimating sand wave migration when no time series of bathymetric data is available during the design of for example a new pipeline or the optimisation of a monitoring and dredging strategy of navigational routes (See also Le Bot et al (2000) and Le Bot (2001)).

To further validate these models, we are interested in additional data. This data should span a couple of years with an interval of about a year. The horizontal positioning error in the data should be less than the migration over the measurement period. Rigid structures like pipelines or other landmarks can increase the accuracy of the horizontal positioning of the data. Furthermore, data on the seabed composition and velocity profiles in the water column to estimate the roughness at the seabed, the magnitude of the slope effect and the remaining parameters in the sediment transport formulation is required. Lastly, depth-averaged values of the (at least) yearly averaged current velocities are required.

Acknowledgments

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