Are 3D morphodynamic simulations without dunes reasonable?

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Abstract
Since the early works of Kennedy, Engelund and Fredsoe the explanation of dune development based on the analysis of the mathematical formulation is known. Morphodynamic models with 3D hydrodynamics have to be unstable in the dune mode, thus giving non-steady results. This is shown with the model SMOR3D. The dune instability and nonlinear development is calculated for the example of a reach of the river Elbe and compared with observed dunes. On the other hand the non-steady dunes may be unwanted by the modeler. Especially if the simulation of the long term behavior is of interest, the dunes are disturbing the computational results very much. The modeler has to choose a smaller morphological time step. Adding that much diffusion, that the instability is eliminated, is questionable, because the 2D alternate bars mode may be damped out as well.

1. Introduction
One very well known explanation of the initiation of dunes has been given by Kennedy, 1964. He analysed the phase shift between the bottom elevation, the current velocity and the water level over a wavy bed. Reynolds, 1965 presented a remarkable instability analyses including the nonlinear treatment of the vertical distribution of velocity and pressure over the wavy bed. Engelund, 1970 succeeded to distinguish between dunes and anti dunes. Fredsoe, 1974 introduced the quite important slope effect. He successfully obtained good agreement between theory and measurements. The well known explanation of the dune instability is the phase lag between transport and bed deformation.
A good explanation can be given by examining the vertical profiles of the streamwise velocity. As sketched in Fig. 1 the wavy bed induces small gradients of the water surface. To obey the continuity equation for the flow the water level is mirroring the bottom elevation. But the hydrostatic pressure gradient has a major influence in the near bottom region.

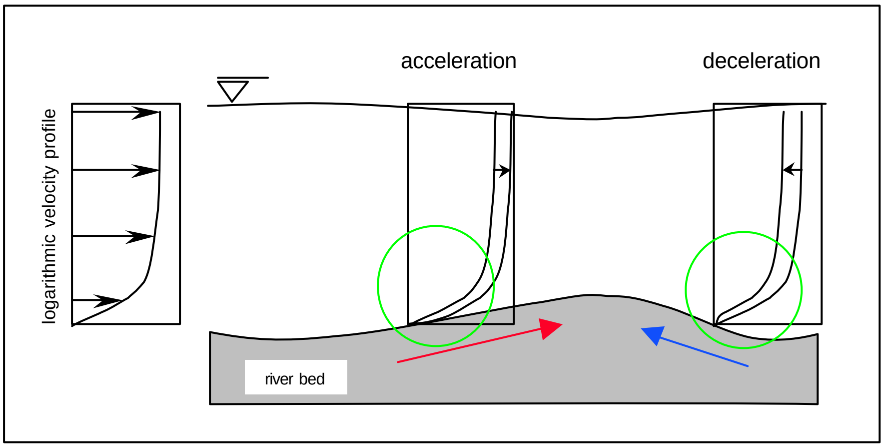

Fig. 1 – Principle sketch of dune instability by considering the velocity profiles and surface elevation. To the left the averaged logarithmic velocity profile is drawn. The local water level gradient influences the near bed velocities.

The mean streamwise velocity is reduced due to bottom friction near the bed. This reduced momentum transport results in a faster and relatively strong reaction of the small bottom velocities on the small pressure gradient induced by the water level. Low current speeds near the bottom are decreased or reversed by a smaller or adverse pressure gradient on the leeside of the just evolving dunes. At the same time the low near bottom velocities are enhanced in front of the dune. This leads to lower transport rates on the leeside and a growing dune. As indicated by the arrows the changes in the velocities enhance the

bottom waves. The main mechanism of the instability is the water level gradient in combination with the differential advection of momentum. Most important for the initiation and development of the dunes are the downslope transport, that influences the dune length, and the 3D flow structure with bottom friction and eddy viscosity.
The nature of this instability is thus explained. A very important property of the dune instability is to be inherent already in the mathematical formulation underlying deterministic morphodynamic models. Thus this explanation of the initiation of dunes is very trustworthy. Otherwise one would put under question all model results that are based on these equations. For the numerical modeler, that has to face a variety of numerical problems and instabilities, this is quite important to know.

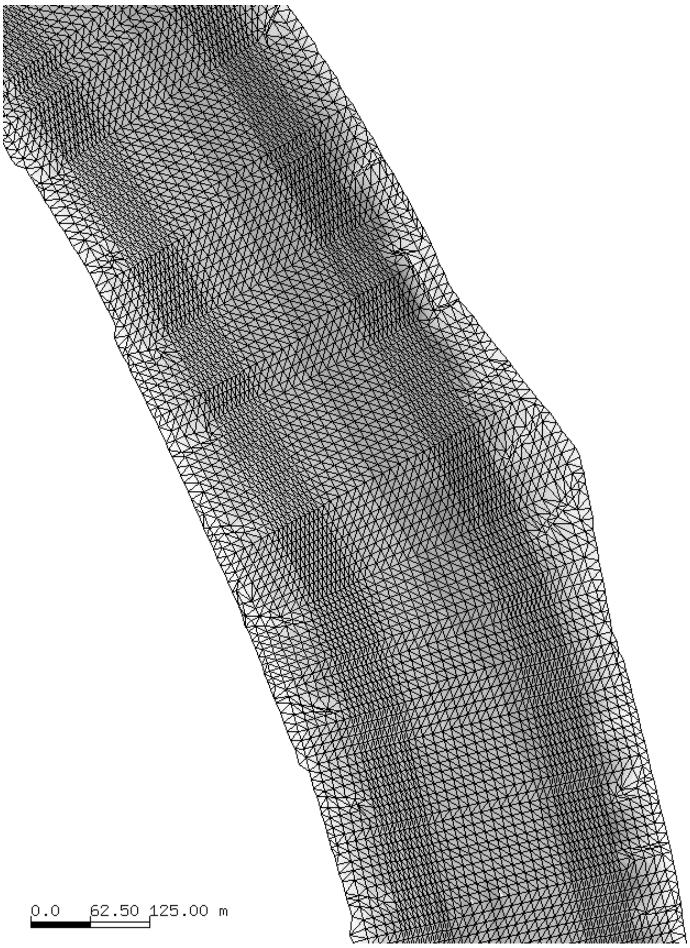

Fig. 2 – Computational grid of the model, with groines and mesh refinement along tip of the groines.

2. Numerical model SMOR3D

2.1 Computational grid

The model calculates the 3D flow field. The vertical direction is discretised into 8 horizontal layers and one bottom layer. The bottom layer is 20cm high, whereas the upper layers are 1 to 2 m high. In contrast to the upper strictly horizontal layers the bottom layer follows the actual river bed. This way the high velocity gradients near the bottom, that are of major importance for the transport calculation, are resolved much better. In the horizontal a triangular FE mesh is used. In Fig. 2 the computational mesh is depicted. It is refined in the shear region along the groin heads. The groins have been build in by refining the mesh appropriately. The grid spacing was about 10 m in the basic version. A refined version has half the grid spacing, i.e. 5 m. With this spacing natural sliding will occur for an elevation difference between neighbor nodes of about 2 m or more. This is more than the observed dune height. Therefore the sliding at the leeside of the dunes is not realistically reproduced, the slopes will be too small. For 1 m high dunes even the separation or flow reversal near the bottom at the leeside will not be good visible, because of the small slope and the coarse vertical resolution. All variables are located in the corner points of the triangles, as in the standard FE formulation. Checker-boarding may occur. Good results have been obtained with an upwind scheme in the sediment transport computation.

2.2 Formulation of the model

The hydrodynamic model uses an explicit leap frog time integration scheme. The Reynolds averaged momentum equations with the hydrostatic pressure assumption and the equation of continuity are solved. For the turbulent mixing in the vertical direction the mixing length approach is used, giving a logarithmic velocity profile. This of course may not give the best solution, in the case of dunes with a desired separation of the flow at the leeside. A more sophisticated model (ε-equation) will otherwise introduce even more problems, especially in the case of a relatively coarse grid.

In this particular model application the transport of momentum in the bottom layer is neglected. This enhances the ability of the model to let the bottom flow velocities change correspondingly to the pressure gradient. Indeed it is very questionable whether the bottom layer that is mostly filled with ripples or other roughness elements allows the transport of momentum as in the case of a flat and smooth bottom. Nevertheless the vertical turbulent momentum exchange counteract this effect. So the near bed velocities are far from being reversed. The effect of the roughness elements on the instability mechanism via the differential advection should be investigated more thoroughly.

The model accounts for bedload sediment transport. The Exner bed-evolution equation is solved. In this application a formulation of the downslope transport is used that adds the weight of a certain moving sediment layer to the shear stress transmitted by the bed velocities. This is equivalent to a formulation used by Fredsoe, 1974.

$$t' = t + C \cdot grad(a) \tag{1}$$

C is a constant determined by the critical bed slope and a is the depth below a horizontal reference level. Using this formulation sliding is simulated with the bedload transport rate when the critical slope is reached. For C the value of $r\, u^{*}_{c}{}^{2}/\tan(\varphi)$ is used, in this case 0.33 Nm. This shear stress is used in the well known bed load equation of Meier-Peter and Müller.

$$q_b(t') = C_{MPM} \cdot ((t' - t_c)/r)^{3/2} \tag{2}$$

The constant C_{MPM} combines the coefficients of this formula. In this calculation no dynamic roughness predictor is used. Also no special coefficients are used to account for the form roughness, because this can easily be done by changing C_{MPM} and u^{*}_{c}. The presented calculation is carried out using only one single grain size.

For the calculation of the bed changes at every triangle node the bed load transport has been balanced within each triangle. Because of problems with the stability of the coupled system within the calculation of the transport rates an upwinding scheme had to be used. Nevertheless mass is conserved exactly. For the groynes a fixed bed level has been prescribed. The groins can be buried but not eroded. This is accomplished by a simple mass conserving algorithm.

At the inflow and outflow boundary the bed level is fixed, allowing for transport at the same time. This ensures, that no disturbances are entering the model domain from upstream. At the outflow boundary a step in the bottom elevation may result in this case, that has not given a reason for instability or errors in the applications.

2.3 Morphodynamic acceleration

For the morphodynamic simulation an acceleration of the morphological time scale is used by multiplying the depth changes by a morphological factor. In this special case the factor was as high as 2000, allowing for a relatively fast computation. This factor has been adjusted so that the changes in each time step are not larger then 10^{-3} m. This way the changes are small and the flow field can react smoothly on the depth changes.

3. Simulation of dunes in the Elbe reach Reststrecke

The study area is located in the lower part of the river Elbe. The mean grain size of the sediment is about 1 mm. In the field dunes and alternate bars are observed. This river each has been studied in a hydraulic scale model experiment and numerical model simulations at the Federal Waterways Engineering Institute in Karlsruhe as well. Both in the field and the scale model dunes develop. The observed wavelength range from 50 to 100 m, the height from 0.5 to 1 m. Beginning with the model area alternate bars are developing intensively, which are rarely met upstream. The basic parameters are the discharge 970 m³/s, that is app. the bed forming dominant discharge and a friction coefficient (λ/8) of 0.0015. The groins are submerged for this discharge.

The data for initial depth distribution is taken from cross-sections that had a distance of about 200 m. Right at the beginning of model simulation these almost invisible disturbances become visible, as dune-like transport bodies develop and move downstream out of the model domain. This is independent of the initiation of dunes thereafter.

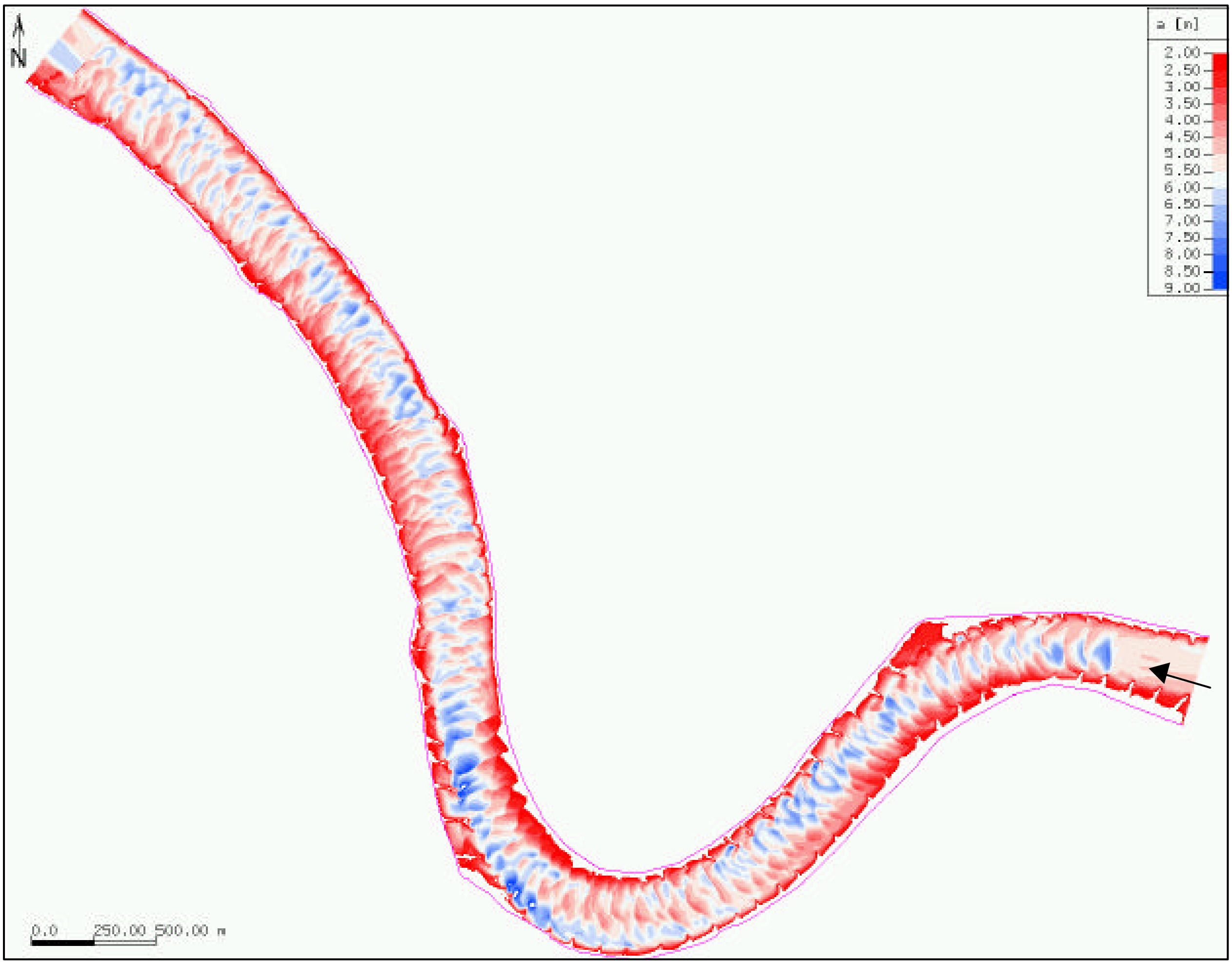

Fig. 3 –Dune bed calculated by morphodynamic model (blue indicates deep)

Instead of becoming smooth, the bed remains unsteady and wavy during the next time. The pattern, that is computed by the numerical model, is depicted in Fig. 3. The flow direction is from right to left, as indicated by the small arrow. The bed features behave like dunes. The entire movable model bed is covered by dunes. They move smoothly downstream between the groines. The propagation speed is about 1.5 m per day for the given situation with constant discharge. Like in the field the propagation velocity of the dunes depend on the size of the dune. Smaller dunes propagate much faster. They may be trapped by the bigger ones. In very long troughs new dunes may appear. The height of the dunes is too large in many places compared the observations. It reaches about 3m. The dunes are threedimensional. Only some of them extend from one bankline to the other. They move even thru the bend and the groin scours. The dunes are highest in the middle of the stream and flatten to the sides. Alternate bars cannot be observed in this picture. They may be hidden by the dunes. Most interesting is the area behind the entrance, where the bed is movable after a short fixed bed section. Here the dunes develop very quick. Already after the distance of about one wavelength the dunes have reached considerable dimensions.

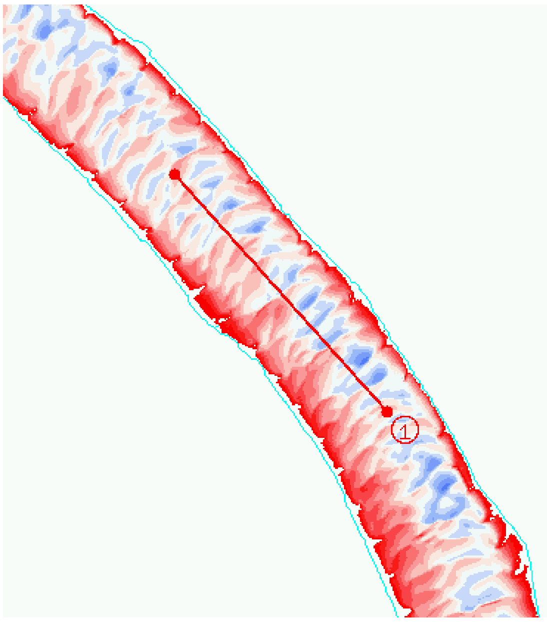

Fig. 5 – Location of the longitudinal depth profile shown in Fig. 6

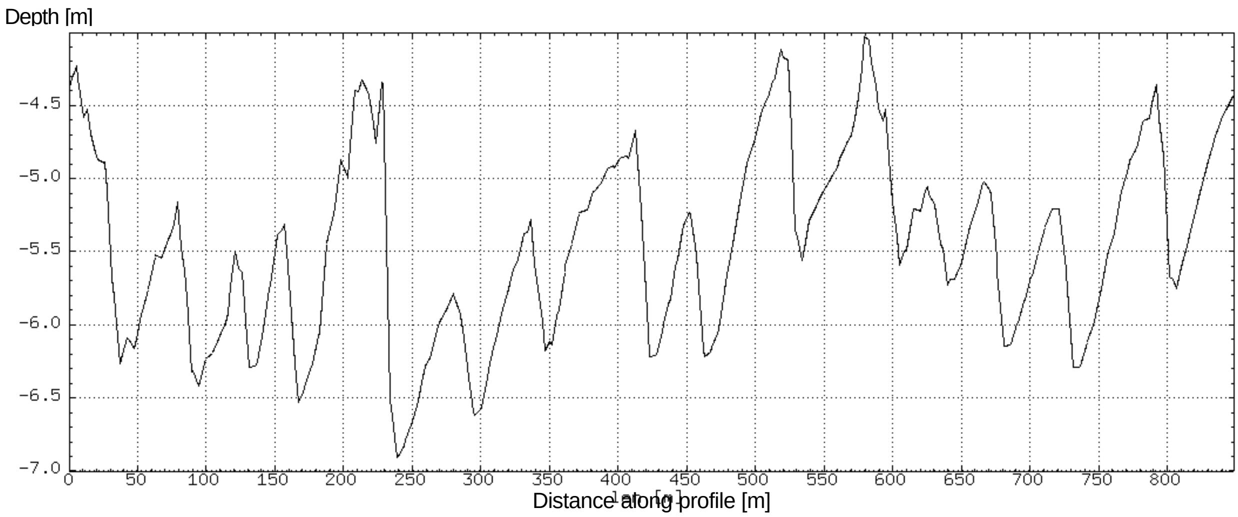

Fig. 6 – Longitudinal depth profile of dune bed along the line given in the small figure.

The longitudinal profile of the dunes is plotted on Fig. 6, at the location shown in Fig. 5. The profile is located right in the center of the river. In the profile the propagation is from left to right. As before the height of the dunes is remarkable with about 1 m in average and 2.5 m for the highest. Also the asymmetry of the dunes is present. The lee side is much steeper then the stoss side. The mean length of the dunes is 57 m. In Fig. 7 3D view of the computed dune geometry is given for the same reach as the longitudinal profile of Fig. 6. The vertical is enhanced by the factor 20. At both sides the bankline and the groins are visible. In the main channel the depth is given in grey, darker in deeper and shallower parts.

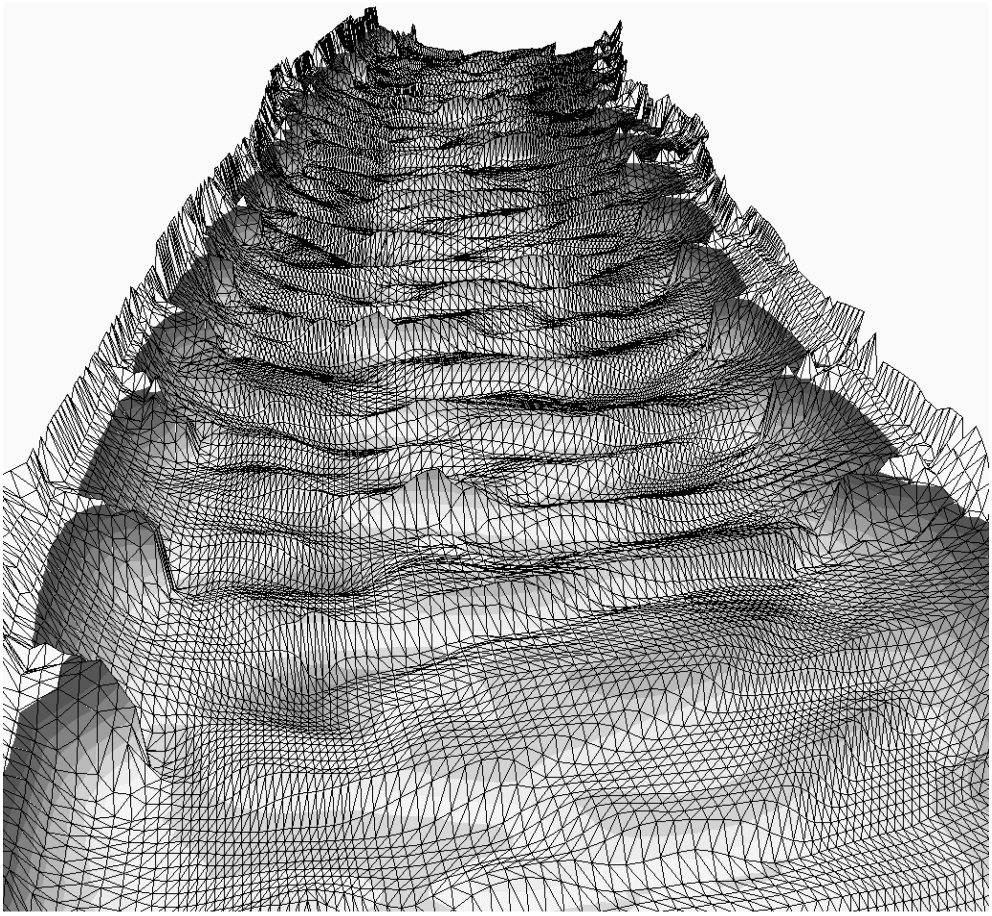

Fig. 7 – 3D view of the dune bed from downstream (vertical is enhanced by a factor 20)

The resulting bed forms are still sensitive against the thickness of the bottom layer of the model, the formulation of the vertical eddy viscosity, bottom friction and slope effect. The grid resolution has still an effect on the results.

4. Conclusion

The presented model results seem to resemble the well known dune instability. Unfortunately almost every scheme introduces a certain phase shift between current field and bed disturbance for the short wavelength of the order of the grid resolution. The resulting instabilities are of course numerical errors! Therefore results, like the presented one, have to be taken with care, especially if the wavelength of the disturbance is near the grid resolution. However if the disturbance is longer than the grid spacing and the short wavelength are damped out, then one can argue that the real dune instability is calculated, as in the presented case.

The dune bed is highly non-steady. Even the bend scours are disturbed by the dunes. Most researchers are interested in the mean bed level. In this case the non-steady dunes are unwanted. The best way is to compute the time mean bed level by averaging the bed levels of the non-stationary simulation with dunes. But this is very time consuming. Therefore it would be an idea to damp the dunes away.

There are several ways to damp the dunes out. One is to add artificial diffusion in the bottom evolution equation. The damping may include the horizontal turbulent momentum exchange and a certain smoothing filter in the water level calculation. In the present case, both corresponds to a diffusion coefficient of 1 m^2/s for a 5 m grid. Another way is to use a large, physically unrealistic lag distance or adaptation length for the bed load transport. Probably there are still other methods to obtain a smooth solution.

The question is: Is it allowed to damp the dunes out?

At the first glance there is nothing to be said against the damping. However it is neither accurate nor very nice. But if we look more thoroughly we find that other instabilities are also damped out in this case. Damping the dunes out we alter the behavior of the solution with respect to other eventually more important instabilities (bedforms) like alternate bars. It is to be expected that the alternate bar mechanism may be damped out by the same amount of diffusion! Therefore the results of a 3D morphodynamic model that produces no dune instability may no longer be trustworthy with respect to other – time mean – effects!

Acknowledgment

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