Geodetic deformation analysis: a new method for the estimation of seabed dynamics

L.L.Dorst[(1)]

[(1)]Hydrographic Service of the Royal Netherlands Navy
P.O.Box 90704; 2509LS The Hague
ll.dorst@mindef.nl

Abstract

The Hydrographic Service of the Royal Netherlands Navy is developing a method to extract seabed dynamics from bathymetric archives. Several gridded soundings of an area and their variances are used to estimate the kind of dynamics in an area, and its size. Hereto, various hypotheses are set up, for analyses per grid point, and for the whole area at once. These hypotheses suppose a static behaviour, an outlying sounding, a trend, or general unstable behaviour, for depth, slope and sand wave phase and amplitude. The combination of hypotheses that fits best to the data is accepted as the most probable. The method is illustrated by an example of a part of a traffic separation lane near the port of Rotterdam.

1. Introduction

The stochastic character of soundings of the seabed makes an analysis on its evolution difficult: even modern multibeam surveys in shoal seas reveal the depth of the sea floor with an accuracy of a few decimetres [International Hydrographic Organization, 1998]. Geodetic techniques like deformation analysis [Caspary, 1987] can deal with stochastic data; geodetic deformation analysis is applied in e.g. terrestrial levelling campaigns. The application of this method to the morphology of the seabed can distinguish between measurement noise and true deformations here as well, including sand wave migration and growth [Dorst, 2003]. An input of representative points on the seabed, several depths for every point and their accuracies are required. Accuracies can e.g. be obtained by the a-priori estimation of variances of every sounding [Hydrographic Department, 1993] using metadata, i.e. a description of the survey: tidal reduction technique, positioning accuracy, etc.

In case of limited seabed coverage this step is followed by geostatistical interpolation [Chilès and Delfiner, 1999], as shown in figure 1. The spatial variability of a stochastic quantity is described by its variogram. This function can be calculated from a data set, and used for the determination of interpolation weights and for an indication of the wavelength of periodic features in the data, like maritime sandwaves. Kriging is an interpolation technique that uses the variogram to estimate interpolated values, and their interpolation accuracy; it is used here to obtain both depths and variances for the representative points, usually in a grid. This paper focuses on the last step of the process of figure 1: the statistical correct comparison of gridded seabed data in time.

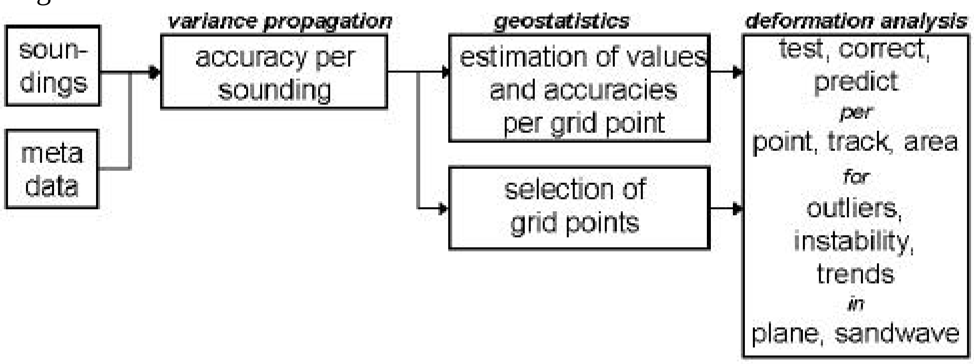

Figure 1 – process for the estimation of seabed dynamics from soundings and its metadata.

2. Method

The method assumes a seabed as sketched in figure 2: a sand wave pattern in one direction, superimposed on a plane. Of course, the seabed has a more complex shape in reality, but one should keep in mind that the aim is not to estimate the shape of the seabed at a single moment, but to detect changes in time. As long as changes are detected well, the model is sufficient. To avoid large errors between the true and the modelled seabed, the area size has to be limited with respect to the scale of the seabed variations.

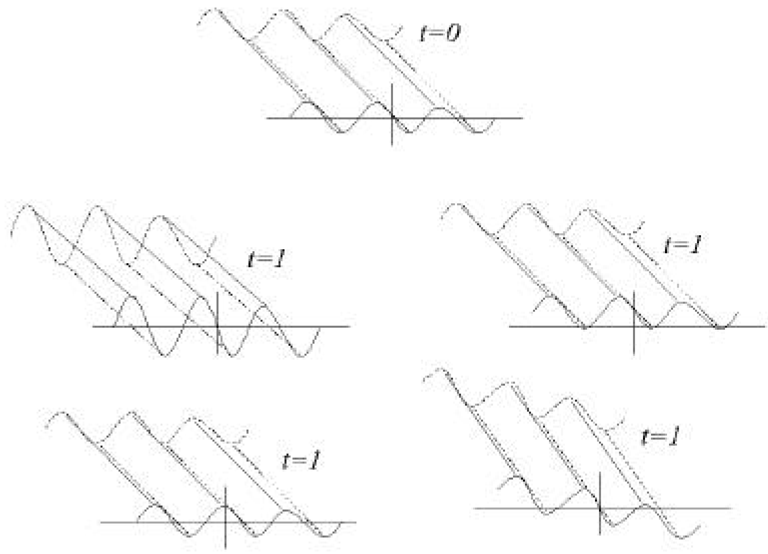

Figure 2 – a modelled shape of the seabed at time *t=0*, and modelled changes at *t=1*: on the left changes in the sand wave, for amplitude and phase; on the right changes in the plane on which the sand wave is modelled, for height and slope.

2.1 Static estimation

This seabed model is expressed as:

$$d_p = \overline{d} + x_p \cdot a_x + y_p \cdot a_y + A \cdot \cos\left(\frac{2p}{l} \cdot x_p + j\right). \qquad (1)$$

This expression relates a depth d_p at the position (x_p, y_p) to the depth of a representative point $\overline{d}$, the slopes a in x- and y-direction, and the amplitude A, phase f and wavelength l of the sand wave pattern. In case the wavelength is assumed constant, and the parameters $u=A\cdot\cos(f)$ and $v=A\cdot\sin(f)$ replace amplitude and phase, the relation between the depths and all the seabed parameters is linear:

$$d_p = \overline{d} + x_p \cdot a_x + y_p \cdot a_y + \cos\left(\frac{2p}{l} \cdot x_p\right) \cdot u - \sin\left(\frac{2p}{l} \cdot x_p\right) \cdot v. \qquad (2)$$

The amplitude and phase can be deduced from u and v: $A = \sqrt{u^2 + v^2}; j = \text{atan}\frac{v}{u}$. Formula (2) is called the 'seabed formula' from here on.

When these depths are measured, the measurement process adds observation noise $\underline{e}$ to the depths: $\underline{d}_p = d_p + \underline{e}_p$. (Stochastic quantities are underlined.) The collection of the unknown seabed parameters of formula 2 in a vector $\boldsymbol{x}$, and their relations with the observations in a model matrix $\boldsymbol{A}$, gives:

$$\underline{\boldsymbol{d}} = \boldsymbol{A}\cdot\boldsymbol{x}+\underline{\boldsymbol{e}}. \qquad (3)$$

$\underline{\boldsymbol{d}}$ is the vector containing all observations $\underline{d}_p$, $\underline{\boldsymbol{e}}$ collects the noise scalars $\underline{e}_p$. The variances s_p^{2} are collected on the main diagonal of covariance matrix $\boldsymbol{Q_d}$. If it is assumed that the observation noise has a normal distribution $N(0,s_p^{2})$, adjustment theory [Teunissen, 2000] states that the Best Linear Unbiased Estimators (BLUE-s) $\underline{\hat{\boldsymbol{x}}}$ of the seabed parameters can be calculated as the least squares solution:

$$\underline{\hat{\boldsymbol{x}}} = \boldsymbol{Q}_{\boldsymbol{x}}\cdot\boldsymbol{A}^{T}\cdot\boldsymbol{Q_d}^{-1}\cdot\underline{\boldsymbol{d}}\,, \qquad (4)$$

having the covariance matrix $\boldsymbol{Q}_{\hat{x}}$ of the BLUE-s defined as:

$$\boldsymbol{Q}_{\hat{x}} = (\boldsymbol{A}^{T}\cdot\boldsymbol{Q_d}^{-1}\cdot\boldsymbol{A})^{-1}. \qquad (5)$$

(An inverse is denoted $^{-1}$, and a transpose ^{T}.) The variances of the seabed parameters can be found on the main diagonal of $\boldsymbol{Q}_{\hat{x}}$.

2.2 Dynamic estimation

If depth observations at p are available from several surveys, $\underline{\boldsymbol{d}}$ is filled with observation vectors $\underline{\boldsymbol{d}}_{\boldsymbol{p}}[s]$ instead of single observations d_p for every p, where s indicates the survey during which an observation at p was made. If the seabed is stable, the vector of seabed parameters does not change. However, in case of an outlying seabed during one survey, the seabed formula should be restated as:

$$\begin{aligned}\boldsymbol{d}_p[s] &= \bar{d} + \boldsymbol{i_s}[s]\cdot D\bar{d}_s + x_p\cdot(a_x + \boldsymbol{i_s}[s]\cdot Da_{x,s}) + y_p\cdot(a_y + \boldsymbol{i_s}[s]\cdot Da_{y,s}) \\ &+ \cos(\frac{2p}{l}\cdot x_p)\cdot(u + \boldsymbol{i_s}[s]\cdot Du_s) - \sin(\frac{2p}{l}\cdot x_p)\cdot(v + \boldsymbol{i_s}[s]\cdot Dv_s).\end{aligned} \qquad (6)$$

The outlying observation during s is identified by assigning the s-th element of $\boldsymbol{i}_s[s]$ the value one, and the other elements the value zero. The ? in front of the new seabed parameters denotes the difference with respect to the general value.

In case of general seabed instability, (2) should be restated as:

$$\begin{aligned}\boldsymbol{d}_p[s] &= \bar{d}_{s_0} + \boldsymbol{?\bar{d}}[s] + x_p\cdot(a_{x,s_0} + \boldsymbol{?a}_x[s]) + y_p\cdot(a_{y,s_0} + \boldsymbol{?a}_y[s]) \\ &+ \cos(\frac{2p}{l}\cdot x_p)\cdot(u_{s_0} + \boldsymbol{?u}[s]) - \sin(\frac{2p}{l}\cdot x_p)\cdot(v_{s_0} + \boldsymbol{?v}[s]).\end{aligned} \qquad (7)$$

s_0 is the reference survey, and the new vectors, denoted by **?**, show the differences with respect to this value. Element s_0 of such a vector is zero.

And in case of a trend, (2) should be restated as:

$$\begin{aligned}\boldsymbol{d}_p[s] &= \bar{d}_{s_0} + \boldsymbol{?t}[s]\cdot\frac{D\bar{d}}{Dt} + x_p\cdot(a_{x,s_0} + \boldsymbol{?t}[s]\cdot\frac{Da_x}{Dt}) + y_p\cdot(a_{y,s_0} + \boldsymbol{?t}[s]\cdot\frac{Da_y}{Dt}) \\ &+ \cos(\frac{2p}{l}\cdot x_p)\cdot(u_{s_0} + \boldsymbol{?t}[s]\cdot\frac{Du}{Dt}) - \sin(\frac{2p}{l}\cdot x_p)\cdot(v_{s_0} + \boldsymbol{?t}[s]\cdot\frac{Dv}{Dt}).\end{aligned} \qquad (8)$$

$\frac{D}{Dt}$ is the speed of the trend for every seabed parameter, and $\boldsymbol{?t}[s]$ is the vector of time differences with regard to the reference survey, $\boldsymbol{?t}[s_0]=0$.

The model extensions in (6), (7) or(8) can be added to equation (3) as [Teunissen, 2001]:

$$\underline{d} = A?x + B_a?\tilde{N}_a + \underline{e}. \qquad (9)$$

B is a matrix that contains the new model parts, and $\tilde{N}$ (pronounced 'nabla') a vector that contains the new seabed parameters. Model (3) is called the null hypothesis H_0, and model (9) the alternative hypothesis H_a for model extension B_a. The new seabed parameters can be divided into two independent sets: the three that describe a movement of the plane, and the two that describe a movement of the sand wave. A hypothesis describing a sand wave movement can therefore be formulated independently of a hypothesis containing a planar movement. Formula (6) can be hypothesised for every survey s; this makes $(s+2)\cdot 2$ hypotheses for possible extensions of the model.

Now, a choice has to be made which hypothesis, or combination of hypotheses, fits the best to the data. This is done by calculating a test statistic T_a for every H_a :

$$\begin{aligned} \underline{T}_a &= \nabla\underline{\hat{d}}_a{}^T \cdot Q_d{}^{-1} \cdot Q_{\hat{e}} \cdot Q_d{}^{-1} \cdot \nabla\underline{\hat{d}}_a ; \\ \nabla\underline{\hat{d}}_a &= B_a \cdot (B_a{}^T \cdot Q_{\hat{r}} \cdot B_a)^{-1} \cdot B_a{}^T \cdot \underline{\hat{r}} ; \\ \underline{\hat{r}} &= Q_d{}^{-1} \cdot (\underline{d} - A \cdot \underline{\hat{x}}); \\ Q_{\hat{r}} &= Q_d{}^{-1} \cdot (Q_d - A \cdot Q_{\hat{x}} \cdot A^T) \cdot Q_d{}^{-1}. \end{aligned} \qquad (10)$$

These test statistics have a $?^2(q,0)$-distribution if H_0 is true, and a $?^2(q,?)$-distribution if H_a is true. q is called the number of degrees of freedom, and $?$ the non-centrality parameter. E.g., q equals three for the planar trend hypotheses, as there are three new model parameters, and two for the sand wave trend hypotheses. The value of the test statistic is compared to a critical value k_a: H_0 is rejected if T_a is larger than k_a – i.e. the differences between surveys are too large to likely be caused by measurement noise, and therefore indicate sea floor dynamics. If H_0 is true however – i.e. the sea floor is stable, still $T_a > k_a$ with $a\%$ probability, due to the misleading influence of the measurement noise. In other words: a is the probability that H_a is accepted if in fact H_0 is true; this probability is called the level of significance. A small a is desirable, but it implicates a large probability $ß$ that H_o is accepted if in fact H_a is true. The optimal choice for k_a therefore depends on the risk of a 'false alarm' compared to the risk of a 'missed alarm'.

If several alternative hypotheses are compared to a null hypothesis, the one that has the largest quotient $T_a/k_{a,a}$ is accepted as true, if this quotient is larger than one. Hereafter, the model is extendend, and the remaining alternative hypotheses are tested again, until the largest quotient is smaller than one. The various hypotheses can be adjusted to each other by tuning the probabilities a: a larger percentage corresponds to a smaller value for k_a and thus to a faster acceptance of an alternative hypothesis, and vice versa.

One might ask the question now: how large should a change in the seabed be to be detected? This question can be answered by calculating minimal detectable biases $|\tilde{N}|$:

$$|\tilde{N}| = \sqrt{\frac{l}{B^T \cdot Q_d{}^{-1} \cdot (Q_d - A \cdot Q_{\hat{x}} \cdot A^T) Q_d{}^{-1} \cdot B}}\,. \qquad (11)$$

Often, $?$ is specified such that the probability $1\text{-}ß$ of detecting a true change with size $|\tilde{N}|$ is 80%. Larger changes can be found more probable, smaller changes less probable.

3. Results

The Royal Netherlands Navy has defined 'critical areas' around the shallowest parts of the Selected Track to the port of Rotterdam. The 'critical area G' in a traffic separation lane near the port of Rotterdam is taken as an example. A Matlab toolbox, developed by the author, has performed the calculations and visualizations of the deformation analysis. For the interpolation, the Kriging toolbox of the University of Quebec was used as well [Gratton]. The representative points are chosen in a grid in the direction of the highest variability, and have a spacing of *100m*. The results of the interpolation of the survey in 2000 are presented in figure 3: depths and their standard deviations. It is obvious that one grid point at the boundary cannot not be estimated as well as the others, because there are only depths available in one direction, and therefore it has a worse standard deviation. Surveys are available for the years 1991, 1995, 1998, 2002 and 2003 as well. The calculated grid points and their standard deviations are comparable. The origin of the grid is *476358E/5750881N* in UTM31-ETRS89 and the azimuth of the *x*-axis is *40°.* The wavelength as estimated from the variogram is *814m*. The chosen levels of significance are: *10%* for trends, *5%* for general instability and *1%* for outliers.

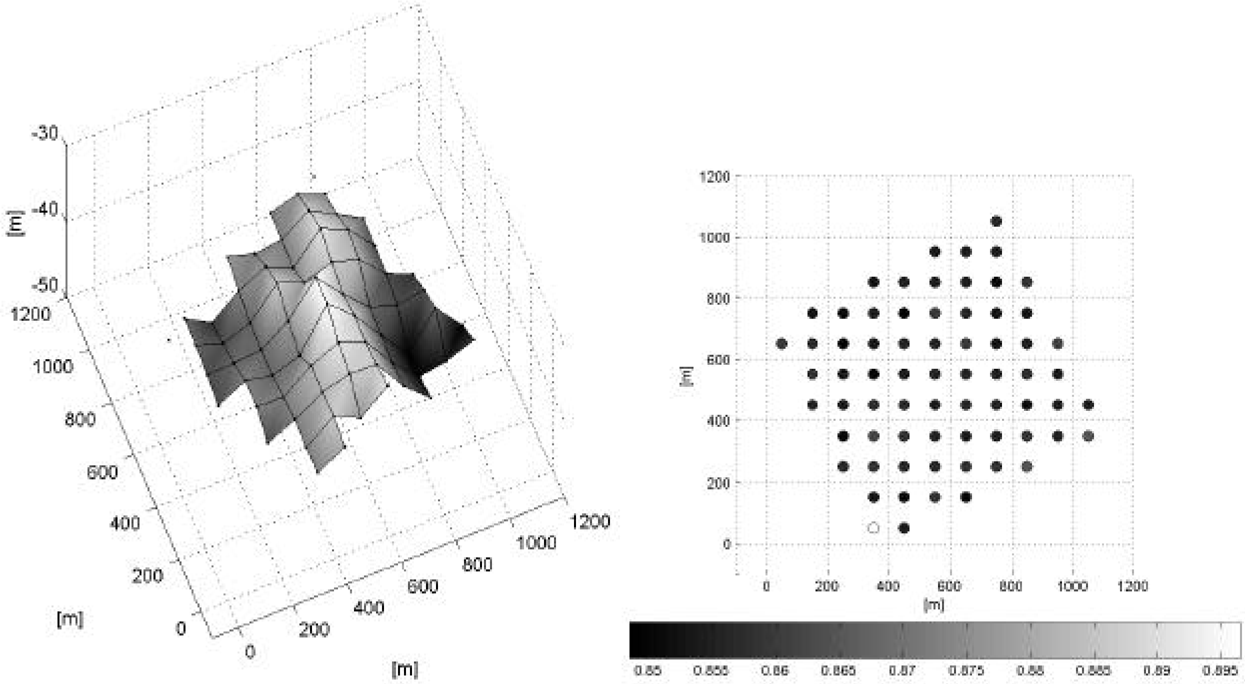

Fig. 3 – gridded depth under MLLWS and their standard deviations [*m*] for the survey in 2000. The vertical axis of the first graph was exaggerated hundred times.

The deformation analysis can be applied to this area as a whole, but also to every grid point individually. The seabed formulae loose all their parts that describe a slope or the sand wave then. Table 1 shows that a lot of change will be invisible when analyzing per grid point. The advantage of this approach is the indication of what is happening inside an area: resolution is gained, but accuracy is lost. Therefore the combination of point analysis and area analysis gives an optimal insight into the dynamics of an area.

type of analysis	**outlying mean depth [*m*]**	**trend in mean depth [*m/yr*]**	**outlying amplitude [*m*]**	**trend in amplitude [*m/yr*]**
per point	*1.87*	*0.16*	-	-
per area	*0.17*	*0.01*	*0.22*	*0.02*

Table 1 – minimal detectable biases that can be detected with *80%* probability for critical area G for levels of significance of *10%* for the trends and *1%* for the outliers.

The outcome of the point analysis is as follows: *45%* of the points shows a trend (*10%* upward; *35%* downward), and *23%* of the points has outlying depths, possibly in combination with a trend. The maximum upward speed is *0.29m* a year, and the maximum downward speed *0.35m* a year. The outliers can mainly be found on the crest, and so can the points that show the largest trends. The trough at the right shows a downward trend, and the back side of the crest an upward trend.

The more precise area analysis shows in the graphs of figure 4 that the points tend to subside with *0.04m/yr* on average. Further, the positive slope in *x*-direction is becoming steeper, and the positive slope in *y*-direction less steep. Positive slopes mean that the depths get larger on average when the coordinates get larger. The trends indicate that the points having large *x*-coordinates subside faster, and the points having small *y*-coordinates as well. This is confirmed by the results of the point analysis. The standard deviations show that the trends in the slopes are hardly relevant however.

The only sand wave dynamics that have been found are an outlying situation in 1991: an amplitude that is *0.1m* larger, and the position of the wave crest is *0.07p rad* further in *x*-direction, what corresponds to 179*m* for this wavelength. A comparison with the standard deviations in the graphs shows that the change in amplitude is not relevant, but the phase change is. A closer look at the shape of the seabed in 1991 confirms the migration of the sand wave pattern.

Note the size of the standard deviations as well: an outlying parameter value cannot be estimated as well as the stable values. Values in a trend can be estimated best in the middle of the time interval.

4. Discussion

Detection of dynamics and the choice for the kind of dynamics depends on the choice of the levels of significance. They can be adjusted for the specific purpose of the analysis: is detection of trends in the seabed behaviour the most important, or maybe the detection of outlying surveys?

The measurement and interpolation noise is thought of as being uncorrelated: the influence of stochastic deviations on one observation is independent of their influence on other observations. In practice, this is not the case. The covariances between the depths, that have a place in the covariance matrix, have been neglected for simplicity. The effect of this neglect should be studied.

The method tries to distinguish measurement noise from seabed dynamics. When the levels of significance are chosen too high, or the variances are chosen too small, measurement noise can still be misinterpreted as dynamics of the seabed too often. Especially in case of one outlying survey, it is impossible to decide from the deformation analysis only whether malfunctioning of equipment has caused an artefact, or that a real seabed disturbance has happened.

Wavelength is deduced from the variogram estimation. More accurate wavelengths can be obtained from estimating it as a BLUE for the seabed formula. A disadvantage is that there is no linear relation between depth and wavelength, so a linearization is necessary, at the cost of computing time. An advantage is the possibility to allow for the same kinds of dynamics for wavelength as for the current five seabed parameters.

A jump in the phase of the sand wave causes an estimation of an amplitude that is too small. This should be kept in mind, especially when defining an area for an analysis. When the variability of the seabed is not periodic enough, no proper wavelength can be estimated anymore, and therefore only a planar analysis can be performed.

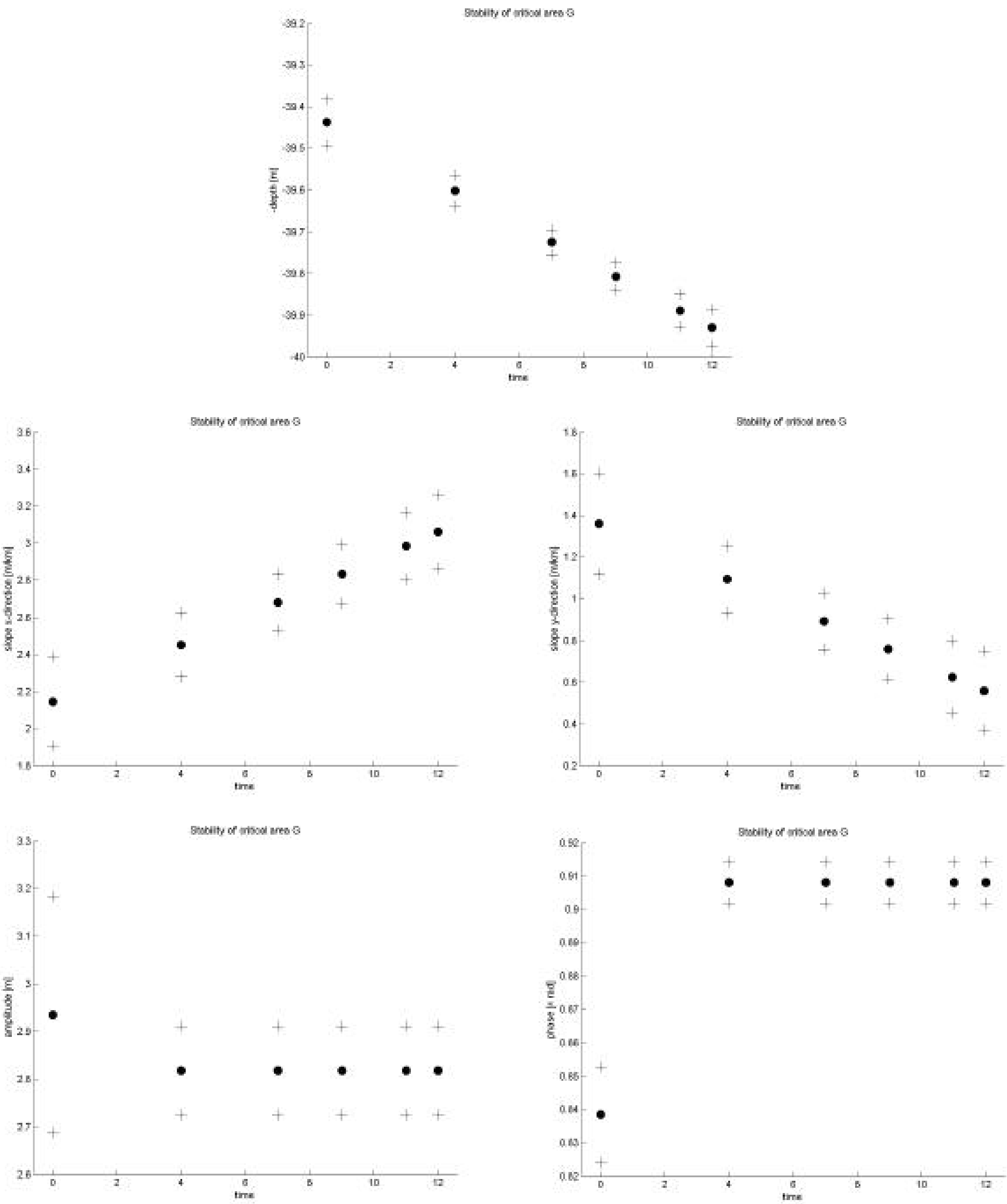

Fig. 4 – dynamics of critical area G.
The dots show the results of area analysis, and the plusses the standard deviations.
Upper and middle row: planar behavior; lower row: sand wave behavior for a wavelength of *814m.*
Time in years since 1991.

Geodetic deformation analysis can be used to perform several other actions on surveys of an area like the usage of the results of the point analysis for the prediction of depths. Not surprisingly, extrapolation of trends results in unlikely depths, as indicated by their high variances. A prediction to the moment of the last survey is very useful however: it provides more accurate depths than the last survey itself. The measurement and interpolation noise can be diminished by including information of past surveys. If no dynamics are discovered at all, every survey shows an image of the seabed that is just as current as the others. The null-prediction then simplifies to an accuracy-weighted average of the surveys, what seems to be the most logical choice.

5. Conclusions

The application of geodetic deformation analysis to a seabed enables the precise estimation of average dynamics of the bed form and depth in an area, and a rough estimation of dynamics within this area, that can e.g. be utilized in research on migration and growth of sand waves. The method is now being compared by the Delft University of Technology to another method for detection and prediction of seabed dynamics, as developed by the North Sea directorate of Rijkswaterstaat [Menting, 2003].

Acknowledgments

The author likes to acknowledge Yves Gratton of the University of Quebec, for sharing the Kriging toolbox, and his colleagues at the Hydrographic Service, for giving direction to this research project.

References

Caspary, W.F., *Concepts of Network and Deformation Analysis*. Monograph 11, Kensington, Australia: University of New South Wales, School of Surveying, 1987.

Chilès, Jean-Paul, and Pierre Delfiner, *Geostatistics, modeling spatial uncertainty*. New York: John Wiley & Sons, 1999.

Dorst, ir L.L., *Zeebodemmonitoring met geostatistiek en deformatieanalyse.* The Hague, The Netherlands: Royal Netherlands Navy, 2003.

Gratton, Yves, www.inrs-eau.uquebec.ca/activites/repertoire/yves_gratton/krig.htm

Hydrographic Department, *The Assessment of the Precision of Soundings.* Professional paper no. 25. London: Ministry of Defense (Navy), 1990.

International Hydrographic Organization, *IHO Standards for Hydrographic Surveys*. Special publication no. 44, 4th edition. Monaco: International Hydrographic Bureau, 1998.

Menting, Peter, *Predicting sea floor changes in the North Sea – Afstudeerplan.* Delft, The Netherlands: Delft University of Technology, 2003.

Teunissen, P.J.G., *Adjustment theory; an introduction*. Series on Mathematical Geodesy and Positioning. Delft, The Netherlands: Delft University Press, 2000.

Teunissen, P.J.G., *Testing theory; an introduction*. Series on Mathematical Geodesy and Positioning. Delft, The Netherlands: Delft University Press 2001.