

## Modelling offshore sand waves: Effect of suspended sediment transport

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### Abstract

In shallow seas, such as the North Sea, sand waves can be observed forming regular bed patterns. Present numerical sand wave models only take bed load transport into account, the most important transport mechanism for sand waves. We expect the suspended sediment transport becomes significant for full amplitude sand waves. Within this study, we investigate the influence of suspended sediment on sand wave dynamics. Firstly, we model the suspended sediment transport with a depth-averaged transport formula. Furthermore, we will employ a vertical sediment concentration profile model. The latter approach is able to describe the phase-lag due to the time difference in pick-up and settling. The suspended load formulation enables modelling influences of wind waves on sediment transport.

### 1. Introduction

Large parts of shallow seas, e.g. the Biscaya Sea or the North Sea, are covered with bed features, forming amazingly regular patterns (Fig. 1). One type of bed features are sand waves. Sand waves have wavelengths of several hundreds of meters and their heights can rise up to 10 m. The sand waves are observed at water depths in the order of 30 m and this means that the height can be considered as significant compared with the water depth. The crests of the sand waves are mostly oriented perpendicular to the direction of the prevailing current. Furthermore, sand waves tend to migrate several metres per year (Idier *et al.*, 2002). Due to their evolution, significant height or migration, sand waves are important for navigation and pipelines purposes. It is necessary to know whether sand waves are able to tend to regain amplitude after dredging, in order to optimise monitoring strategies and dredging policies (Knaapen and Hulscher, 2002). If pipelines are exposed due to sand wave migration, knowledge on migration will help in pipeline policy (Morelissen *et al.*, 2003)

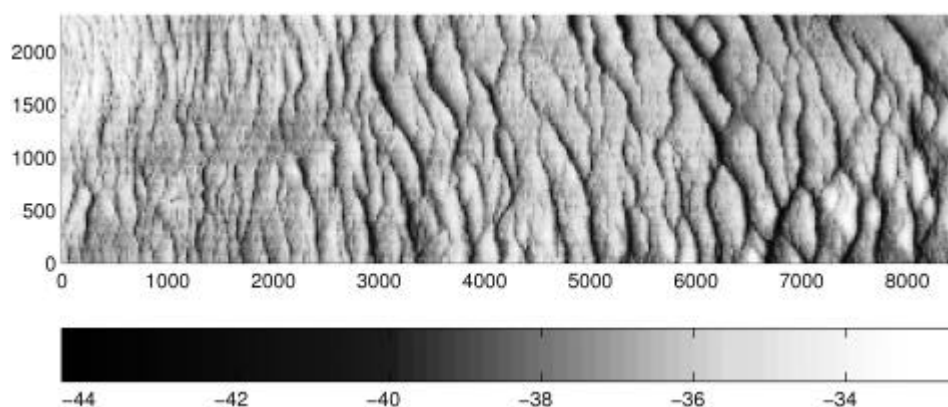


Fig. 1 - Observed sand wave patterns in the North Sea near the Eurogeul (horizontal distance in meters and the colorbar denotes sea bed level below mean sea level in meters). (Courtesy of Rijkswaterstaat, North Sea Directorate (Knaapen and Hulscher, 2002))

Fredsøe and Deigaard (1992) related the equilibrium sand wave height, length and migration to the sediment transport parameters. With this approach it was not possible to describe growth of sand waves as the time variable was eliminated. Hulscher (1996) and Komarova and Hulscher (2000) investigated the time-dependent behaviour of sand waves, with the help of a linear stability analysis applied to a morphological model. This model describes the interaction between vertically varying water movement, bed-load sediment transport and an erodible bed in a shallow sea. This analysis shows that sand waves can evolve as free instabilities of the seabed-water system. Furthermore, the existence of sand waves at certain locations in for instance the North Sea can be predicted with these models (Hulscher and Van den Brink, 1999). Németh *et al.* (2002) extended the research by introducing an asymmetry in the water motion. Németh *et al.* (2002) added a unidirectional steady flow ( $M_0$ ) to the tidal motion ( $M_2$ ). The results show a net migration of the sand waves in the direction of the steady flow. These migration rates agree with theoretical and empirical values found in the literature (see (Allen, 1980) and (Lanckneus and De Moor, 1991)). Recently, Besio and Blondeaux (2003) extended a linear stability analysis with a suspended sediment transport formulation. This stability analysis showed that especially in case of fine sediment the suspended sediment transport changes the initial behaviour significantly. Including a suspended sediment transport formulation increases the growth rate and bed forms tend to have shorter wavelengths with respect to bed load transport only.

The limitation of a linear stability analysis is that its validity is limited to small-amplitude sand waves only. Komarova and Newell (2000) investigated the non-linear behaviour of tidal sand waves and sand banks. This analysis coupled amplitudes of sand waves to mean bed level dynamics. The results only describe regular patterns and cannot describe migration, irregularities and stochastic variations. Németh *et al.* (2003) developed a numerical simulation model to enable the investigation of the finite amplitude behaviour of sand waves (Fig. 2). With the numerical simulation model it is possible to predict initial growth rates and describe the finite amplitude behaviour of the sand waves as well.

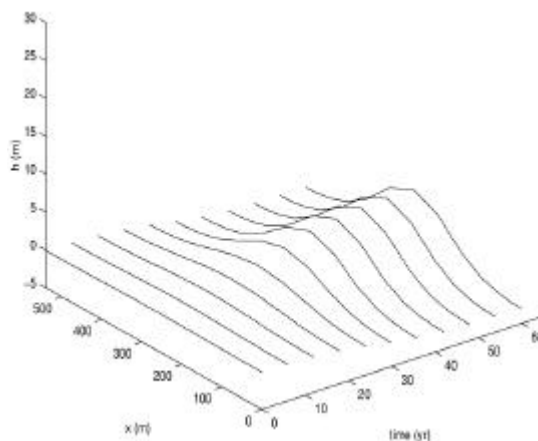


Fig. 2 – Sand wave evolution for typical North Sea conditions with bed load transport. The initial amplitude of the imposed sand wave is 0.025 m. It takes about 20 years to evolve from 10% to 90% of the saturation height (Németh and Hulscher, 2003).

The general goal of this study is to investigate the effect of suspended sediment transport on the behaviour of finite amplitude sand wave evolution. To enable this analysis, we will start with the numerical simulation model by Németh *et al.* (2003) extending the bed load transport formulation with simple suspended sediment transport formulations. Firstly, we aim at testing quantitatively the influences of suspended sediment transport. Hereto, we will use a depth-averaged suspended sediment transport formulation and investigate on the selection of the linearly most unstable mode, compared with only bed load transport. Next, we will investigate the effect of suspended sediment transport on the evolution of sand waves with the simulation model by comparing the saturation heights of the sand waves. We expect

suspended sediment transport to have a stabilizing effect on sand wave evolution and that this effect will be stronger for shallower waters (~20 m), where we assume suspended sediment transport to play a more important role. Furthermore, we expect during storms more suspended sediment transport than during calm weather.

## 2. Modelling sediment transport

### 2.1 Hydrodynamics

The Coriolis force only slightly affects sand waves (Hulscher, 1996). Therefore, the numerical simulation is based on the two-dimensional vertical (2DV) shallow water equations with a free surface:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -g \frac{\partial \zeta}{\partial x} + A_v \frac{\partial}{\partial z} \left( \frac{\partial u}{\partial z} \right) \quad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \quad (2)$$

Here  $u$  and  $w$  are respectively the velocities in  $x$ - and  $z$ -directions. The gravitational acceleration is denoted by  $g$  and  $A_v$  is the vertical eddy viscosity. Finally,  $\zeta$  is the free surface elevation and  $t$  is the time (Fig. 3).

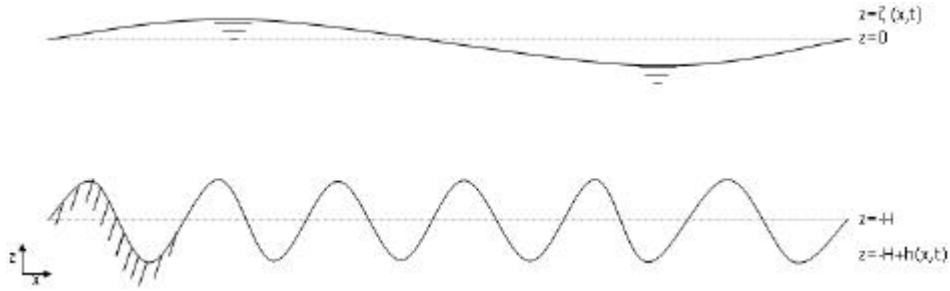


Fig. 3 – Sketch of the dimensional model definition. The horizontal and vertical direction are given by respectively  $x$  and  $z$ . The free surface is defined by  $\zeta$  relative to  $z=0$  and  $h$  is bottom level with respect to the average bottom level ( $z=-H$ ).

In the horizontal plane the boundaries are assumed to have the same properties on both sides of the sand wave. In order to do so, we assume periodicity in the sand wave field. This means that we assume only one type of sand wave to be present in the sand wave field. The boundaries at the free surface are defined as follows:

$$\mathbf{t}_w = A_v \left( \frac{\partial u}{\partial z} \right)_{z=surface} ; \quad \frac{\partial \zeta}{\partial t} + u \frac{\partial \zeta}{\partial x} = w \quad (3)$$

The first equation involves the influence of wind-induced shear stress ( $\mathbf{t}_w$ ) at the surface. The wind-induced shear stress results in a current besides the current induced by the tide. This does not take the influence of wind waves on sediment transport into account. Finally, the boundaries at the bottom are described with a partial slip condition ( $S$  is the resistance parameter and  $h$  is the seabed level):

$$\mathbf{t}_b = A_v \left( \frac{\partial u}{\partial z} \right)_{z=bottom} = Su; \quad \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} = w \quad (4)$$

## 2.2 Sediment transport

The next step is to couple the flow model to the sediment transport formulations. Bed load transport, is a dominant factor in the behaviour of sand waves. Therefore, available models neglect the influence of suspended sediment transport on the evolution of sand waves. The bed load transport can be modelled directly using the bottom shear stress resulting from the flow model (Komarova and Hulscher, 200):

$$Q_b = a |t_b|^b \left[ t_b - I \frac{\partial h}{\partial x} \right] \text{ with } I = I_1 - I_2 |t_b| \quad (5)$$

Here  $Q_b$  is the volumetric sediment transport based on bed load,  $b$  the power of transport and  $a$  the sediment transport parameter.  $I$  is a parameter compensating for gravity influence, taking into account that sediment moves more easily downhill than uphill. And  $t_b$  is the bottom shear stress neglecting the critical shear stress, the shear stress necessary to bring sediment in motion (see eq. 4).

The model with only bed load transport is able to describe the initial growth rates comparable to the results of linear stability analyses (see (Hulscher, 1996) and (Németh *et al.*, 2001)). It also describes intermediate term sand wave behaviour (Fig. 2), where we expect suspended sediment to be no longer negligible. Therefore, we add suspension transport to the simulation model (Németh, 2003). In order to add suspended transport, we investigate, in line with the bed load transport formulation, a depth-averaged transport formula (Walgreen *et al.*, 2002) based on the model of Bailard (1981):

$$Q_s = \frac{n_s}{1-p} \left[ 4 \frac{H}{H_0} \langle |u_w|^3 \rangle u - I_s \langle |u_w|^5 \rangle \frac{\partial h}{\partial x} \right] \quad (6)$$

Here are  $H$  and  $H_0$  respectively the water depth and the undisturbed water depth. The gravitational influence on the movement of the particles is described by  $I_s$ , where  $I_s$  is based on the fall velocity. The depth averaged tidal current is represented by  $u$  and the wave-orbital velocity by  $u_w$ . The wind wave-orbital velocity needs to be compensated for the changing water depth:

$$u_w = u_{w0} \left( \frac{H_0}{H} \right)^{m/2} \quad (7)$$

As the size of the suspended sediment transport is larger on shallower water, thus on the top of sand waves, suspended sediment transport is expect to have a dampening effect on the height of sand waves. In the depth-averaged transport formula (see eq. 6), the effect of phase lag is neglected. This phase lag results from a time span between stirring up and settling of sediment. In order to model this phase lag, we introduce a more advanced model based on a vertical sediment concentration profile. In order to model the vertical sediment concentration profile, we make use of an advection-diffusion equation (De Swart and Falques, 1998):

$$\frac{\partial C_s}{\partial t} = -u \frac{\partial C_s}{\partial x} + K_h \frac{\partial^2 C_s}{\partial x^2} + K_v \frac{\partial^2 C_s}{\partial z^2} + (w_s - w) \frac{\partial C_s}{\partial z} \quad (8)$$

Here  $C_s$  is the concentration of suspended sediment. To solve this equation we need additional boundary conditions. For the boundaries in the horizontal plane we assume in line with the flow model periodicity, in the vertical plane we need two conditions for the free water surface and the bottom. Both conditions are modelled with the erosion-deposition flux normal to the boundary,  $F_*$ :

$$F_* = -n \cdot \left( K_h \frac{\partial^2 C_s}{\partial x^2} + K_v \frac{\partial^2 C_s}{\partial z^2} + w_s \frac{\partial C_s}{\partial z} \right) \quad (9)$$

At the non-erodible boundary (e.g. the free surface)  $F_* = 0$ , for the erodible boundary the flux describes the stirring up of the sediment particles:

$$F_* = w_s (c_a - c_o) \quad (10)$$

Here is  $c_a$  the so-called reference concentration. It describes the effect that if stirring up takes place, the concentration near the bottom will tend to an equilibrium concentration ( $c_a$ ) where settling down equals to

the rate of erosion. The concentration near the bottom ( $c_0$ ) can be estimated with the depth-averaged concentration and the equilibrium concentration defined by the bottom shear stress (De Swart and Falques, 1998):

$$c_0 \equiv \bar{C} \frac{w_s}{K_v}; \quad c_a = r_s (1-p) \left[ \frac{g_o s}{1 + g_o s} \right] \quad (11)$$

Here is  $\rho_s$  the density of sediment particles and  $p$  accounts for the porosity of the bottom.  $\rho_o$  is a constant accounting for the rate of erosion as an influence of shear stress and  $s$  describes the relative shear stress compared to the critical shear stress of the bottom:

$$s = \frac{t_b - t_c}{t_c}; \quad (12)$$

### 2.3 Seabed evolution

The last step is to couple the sediment transport models to the bottom evolution. This is done by assuming a zero net inflow of the combined sediment transports resulting in the following sediment balance (Németh, 2003):

$$-\frac{\partial h}{\partial t} = \frac{\partial Q_b}{\partial x} + d \frac{\partial Q_s}{\partial x} + (1-d) \frac{F_*}{r_s (1-p)} \quad (16)$$

Where  $d$  stands for the choice of suspended sediment transport mechanism,  $d=1$  results in a depth averaged suspended sediment transport computation and with  $d=0$  the suspended sediment flux is computed with the vertical sediment concentration profile.

The model is periodic in the horizontal direction and non-periodic in the vertical direction. A sigma-coordinate transformation is used to deal with the variations in the vertical of the water level and the position of the seabed. For the spatial discretization in both directions a spectral (Chebyshev-Gauss-Lobatto) collocation method is used (Canuto, 1988). The spectral discretization implies a variable spatial grid size. For example, with 15 grid points on one sand wave, the grid sizes varies from 1 to 11% of the wavelength, with the largest grid size in the middle of the domain. For the temporal discretization an implicit time integrator is used.

The model can be split up in three parts, the flow model, the calculation of sediment transport and the seabed evolution. The flow model has to be calculated on a tidal timescale whereas the sediment transport and the bed evolution can be calculated on a larger timescale with tidally averaged values (Németh, 2003). Furthermore, as suspended sediment transport will only take place close to the bottom, modelling of the vertical sediment concentration profile, requires a high vertical density of grid cells close to the bed. In order to deal with the different spatial and time scales, the three parts are solved separately. Firstly, the flow is modelled and with the results the bed load and suspended load transport is calculated resulting in a bed level change (Fig. 4).

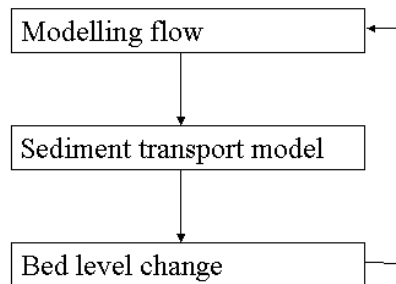


Fig. 4 - Schematic view of steps in computational model

### 3. Results

In order to investigate the influence of the suspended sediment transport we compared the default set up (Tab. 1) with the combined bed load and depth-averaged suspended load formulation (Fig. 5, 6 and 7).

Tab. 1 - Input parameters of default set up (only bed load transport).

Parameter	Symbol	Value
Wave length	L	400 m
Water depth	H	25 m
Vertical constant viscosity	$A_v$	0.01 m <sup>2</sup> /s
Sediment transport parameter	a	0.3 s/m <sup>2</sup>
Bottom gradient parameter	$\gamma_1$	0.006 s/m <sup>2</sup>
	$\gamma_2$	3.33 [-]

In the suspended load formulation we varied the wind wave speed in order to test the sensitivity of the transport formula to the different weather conditions. We took for the wind wave speed 1 m/s (representing a wave height of 1.5 m) and a speed of 1.5 m/s (wave height of 4 m). The values of the input for the suspended sediment transport formulation (Tab. 2) are based on Walgreen *et al.* (2002).

Tab. 2 - Input parameters for suspended load following Walgreen et al. (2002).

Parameter	Symbol	Value
Wind wave speed	$U_{w0}$	1 and 1.5 m/s
Type of suspended transport	d	1 [-]
Power (depth-speed relation)	m	1.6 [-]
Viscosity	$\gamma_s$	5.6*10 <sup>-4</sup> s <sup>3</sup> /m <sup>2</sup>
Bed slope parameter	$\gamma_s$	0.3 s/m

The suspended sediment transport does not have much influence on the behaviour of sand waves (Fig. 5). The fast growing mode is not changing significantly, but the initial growth rate is a bit smaller in case of the high wave conditions.

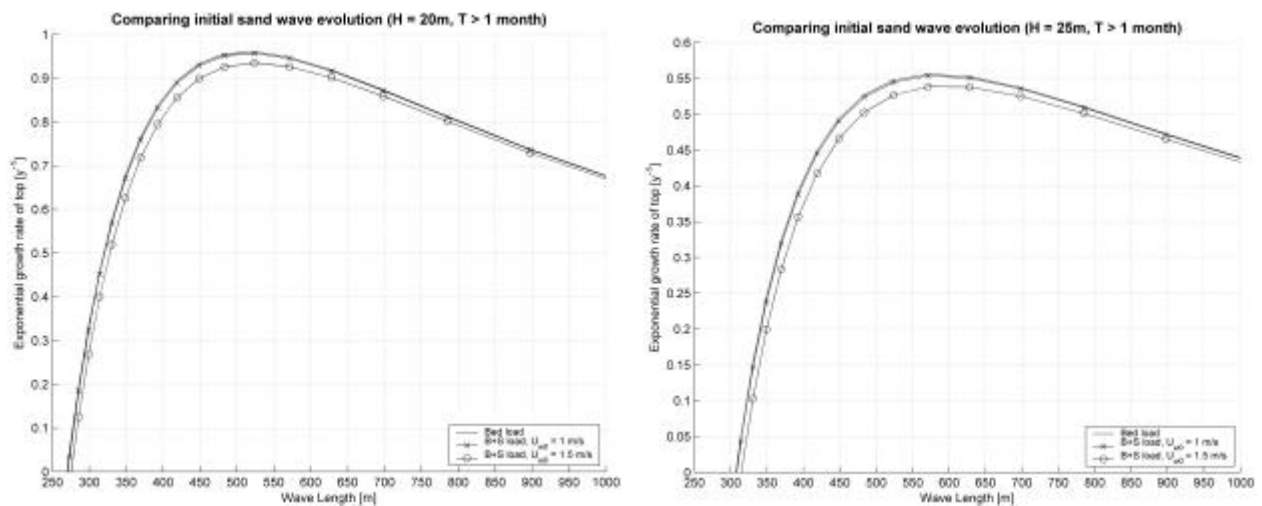


Fig. 5 - Comparing initial sand wave evolution for 2 water depths and 2 wind wave speeds.

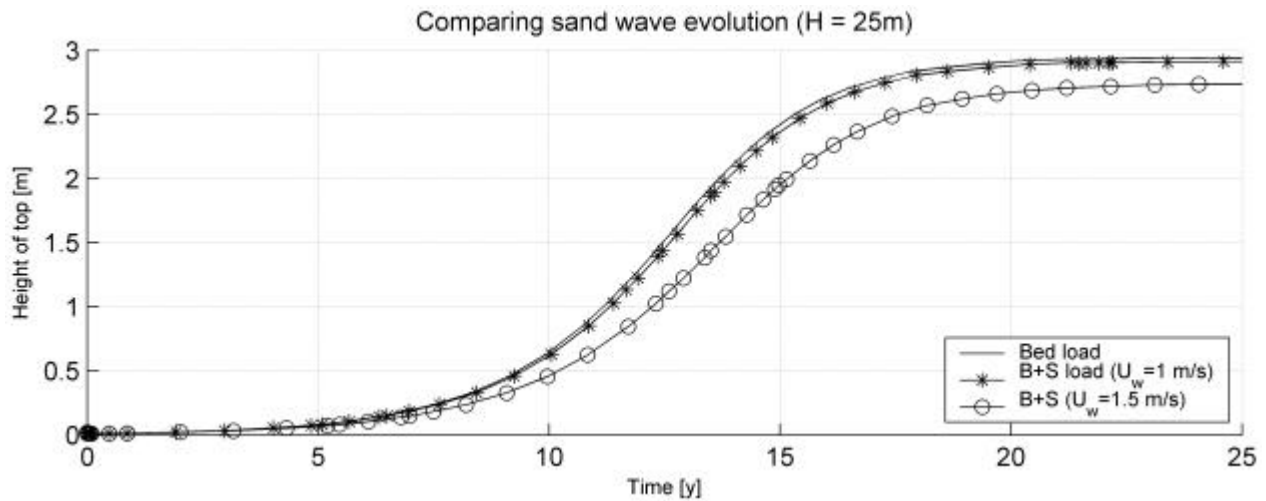


Fig. 6 - Comparing sand wave evolution for 2 wind wave speeds with bed load only, water depth is 25m.

Looking at the long term evolution we see that, with a saturation height of about 3 meters, the differences after 25 years are about 1% for suspended sediment transport under average wind wave conditions (Fig. 6). But, suspended sediment transport with increased wind wave speed results in a significantly lowering of the saturation height (about 7%). Decreasing the water depth gives somewhat the same results (Fig. 7).

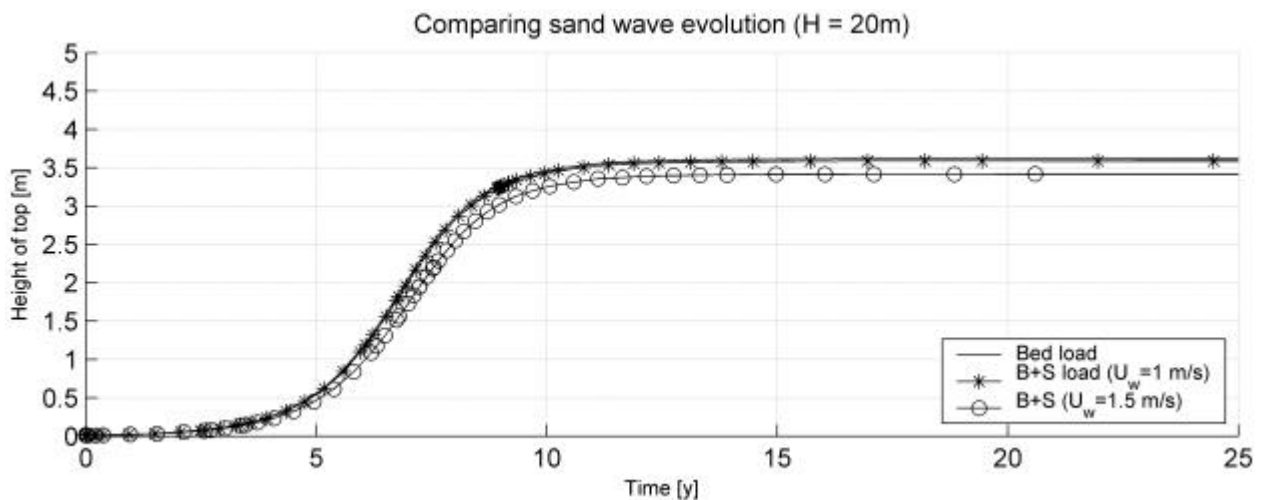


Fig. 6 - Comparing sand wave evolution for 2 wind wave speeds with bed load only, water depth is 20m.

#### 4. Discussion

Suspended sediment transport changes the initial growth rates a bit and the fast growing mode does not change at all. But, on the long term suspended sediment transport has a stabilizing effect on sand waves, resulting in lower saturation heights compared to the situation taking only bed load transport into account. This especially holds for large wind wave conditions. The depth-averaged formula only decreases the gradient of the total sediment transport, as it is opposite to the gradient of the bed load transport. The phase-lag, due to the fact that settling of suspended sediment particles takes time, has not been taken into account in the depth-averaged formulation.

The combined bed load and suspended load transport formulation will be the base for future work. With suspended sediment incorporated in the sediment transport formulation we are now able to take wind waves into account. We expect that during heavy weather conditions more sediment will come into

suspension and will be transported. After the storm is over, the sediment is able to settle again, but at a different location than the pick-up point. Therefore, we have to make use of the suspended sediment transport based on the vertical sediment concentration profile.

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