Sediment transport rates over bedforms: observations and model predictions

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Abstract
Hydrodynamic data and sea bed observations were collected from an autonomous multi-sensor tripod (STABLE) in the vicinity of the Broken Bank, off the Norfolk coast, UK. The performance of 10 existing predictive sediment transport formulae are compared against sediment transport rates inferred from observed ripple migration rates. A refined bottom roughness model proposed by Powell et al. (in this volume) is adopted here:

$$\log(k_s) = 4.09 h \cos q_{cr} - 3.42$$

Wide variation between the sediment transport rate results is evident. A recalibrated formula is suggested, similar to those proposed by Soulsby (1993), Williams $et\ al.$ (1989) and Hardisty (1983).

$$I_b = B g (1 - \frac{r}{r_s})(t_b - t_{cr}) U_{100}$$

This formula offers considerable improvement over those proposed previously.

Introduction

Sediment transport in the marine environment is the result usually of both steady current and wave action. However, before accurate prediction of bedload transport rates can be attempted, detailed knowledge of wave-current interaction and the shear stress distribution in the bottom boundary layer must be known. Estimation of the bed shear stress in combined wave-current flows is difficult, due to the complex nature of flow processes in the near-bed region, and the need to specify accurately the boundary roughness.

Once the bed shear stress has been derived, further variation is encountered as different bedload transport equations often produce widely differing values from the same input conditions (see e.g. Heathershaw, 1981; Pattiaratchi and Collins, 1985). None have been developed for use in the sea under oscillatory tidal currents; most are based on steady-flow concepts and are unable to account intrinsically for the effects of waves. A recognised way of improving the accuracy of sediment transport predictions is to compare the different formulations against accurate field measurements. Accurate measurements of bedload transport are, however, difficult to obtain. Photographing the sea bed, for the determination of ripple size characteristics and migration rates, potentially provides a non-intrusive method of determining bedload transport rates.

Bedload sediment transport rates, using 10 widely-reported formulae, have been determined using the predicted shear stresses presented by Powell et al. (this volume), integrated over a wave cycle. These predicted rates have been compared with the sediment transport rates inferred from observed ripple migration rates.

1. Data Collection

An autonomous multi-sensor tripod, STABLE (Sediment Transport And Boundary Layer Equipment, Humphery, 1987), was used to obtain hydrodynamic data and sea bed observation. The tripod was deployed for a period of approximately 3 days on the southwestern side of Broken Bank, off the Norfolk coast, UK. The mean water depth was approximately 29 m and the sea bed sediment consisted of well sorted sand (mean grain size of 0.22 mm). Currents were measured at heights of 41 and 80 cm above the bed and a pressure sensor was used to estimate the mean water depth. Photographs of the sea bed were obtained using a 35 mm camera, located at 1.25 m above the bed and looking vertically downwards. Further details concerning data collection and shear stress calculation is presented by Powell at el. (this volume).

2. Methods of data analysis
Predicted bedload transport rates
Estimates of bedload transport rates have been based upon shear stress values (τ_c and τ_w), as described by Powell et al. (this volume). Bed shear stresses and friction velocities determined from flow data recorded 0.8 m above the bed are, however, associated with bed roughness due to both skin friction and form drag. Hence, an attempt has been made to identify the contribution of each roughness component (skin friction and form drag), to the total drag.

For transport applications, the force applied to individual particles should be used i.e. the total shear stress acting on the rippled bed should not be used, only that proportion which represents skin friction. Friction velocities (u_*) obtained using the boundary layer model of Sleath (1991) are subject to both skin friction and form drag; sediment transport rates based on these values would be expected to overestimate the observed rates. Using the boundary layer model of Christoffersen and Jonsson (1985), friction velocities based upon recorded flow data (at 0.8 m above the bed) and k_s equal to the grain diameter and 'measured' bed roughness (Powell et al., this volume), have been determined. Using these friction velocities and assuming a logarithmic velocity profile, the flow velocities at the height of the wave boundary layer has been evaluated. Below this, a velocity profile has been assumed to exist (Chris and Caldwell, 1982; Kemp and Simon, 1983). This latter logarithmic profile is dependant only upon the grain roughness (Figure 1). These velocities (and heights) were used then to obtain friction velocities with $k_s=D$; hence, they are dependent only upon grain roughness (G. Madsen, personal communication). The friction velocity obtained is, in this way, independant of the influence of form drag of the bedforms.

The proportion of the shear stress used in overcoming form drag can be determined. The skin friction component of the total shear stress is given by:

$$\frac{u_{*(n)}}{u_{*(n+1)}} \tag{1}$$

where $u_{*(n)}$ is the friction velocity acting on the sediment grains alone; and $u_{*(n+1)}$ is the friction velocity resulting from the roughness of the sea bed sediment and bedforms.

Transport under the combined action of waves and currents
In the marine environment, where waves and currents are superimposed, the forces acting on the sea bed sediment can be separated into those resulting from: (i) a steady flow (tidal current) component (τ_c); and (ii) an oscillatory (wave) component (τ_w), which is time-varying with a period of oscillation equivalent to the period of the waves. Non-linear interaction occurs between these forces. The total bed shear stress (τ_b) is given by the vector sum of the two components, so that the instantaneous bed shear stress is given by:

$$\vec{t}_b(t) = \left(t_x(t), t_y(t) \right) \tag{2}$$

where

$$t_x(t) = |t_w| \cos\left(\frac{2pt}{T} + f \right) + |t_c| \cos(f_{wc}) \tag{3}$$

$$t_y(t) = |t_c| \sin(f_{wc}) \tag{4}$$

where, τ_x and τ_y are instantaneous bottom stress components in directions parallel and perpendicular to the wave propagation, respectively; T is the wave period; ϕ is the phase difference between the orbital velocity and wave shear stress; and ϕ_{wc} is the angle between the waves and currents. Depending upon the relative magnitude of waves and currents, the resultant shear stress can be variable (in terms of magnitude and direction) during the wave cycle. In order to account for such variability, instantaneous sediment transport rates can be calculated for each (orthogonal) direction:

$$I_b(t) = F\left(|t_b(t)|, t_{cr} \right) \frac{t_b(t)}{|t_b(t)|} \tag{5}$$

where F is the function associating bottom shear stress to bedload transport rates.

For the estimation of bedload transport rates, the assumption that z_o is equal to the physical roughness (i.e not

enhanced by waves) is reasonable i.e. the difference in roughness is not very large. If the transport of material in suspension was to be included, then even small differences in the roughness parameter could introduce large errors in the derivation of loads; this is because of high suspended sediment concentrations present near the seabed.

Another assumption made in the calculation, valid at least for shallow coastal waters, is that the wave-induced velocity remains constant from the top of the wave boundary layer (δ_w) to the mean water surface. No estimation of the vertical distribution of velocity inside the wave boundary layer is considered, since this is very thin (i.e. the order of 1 to 2 cm); this, therefore, does not affect the results of the transport formulae predictions.
The instantaneous wave-induced orbital velocity is given by:

$$u(t) = U_{rms} \cos\left(\frac{2pt}{T}\right) \tag{6}$$

The instantaneous depth-integrated wave-current velocities, at various incidents along the direction of wave propagation (x), and perpendicular to the wave propagation (y), $\hat{U}_x(t)$ and $\hat{U}_y(t)$ respectively, are given by:

$$\hat{U}_x(t) = \hat{U}\cos(f_{wc}) + U_{rms}\cos\left(\frac{2pt}{T}\right)$$
$$\hat{U}_y(t) = \hat{U}\sin(f_{wc}) = \text{constant} \tag{7}$$

As the x-direction is parallel to that of wave propagation, the combined velocity in the y direction is constant, dependent only upon the tidal current velocity. The x and y components of instantaneous sediment transport rates are presented as described above, for equations where I_b is a function of bottom shear stress.

The mean (immersed weight) sediment transport rate, over a wave period is given then for each direction, by:

$$I_{bx} = \int_{t=0}^{t=T} I_{bx}(t)dt \tag{8}$$

$$I_{by} = I_{by}(t) = \text{constant} \tag{9}$$

Hence, the total sediment transport rate (I_b) and direction (ϕ_{Ib}), with reference to the wave direction (x-axis), are given by:

$$I_b = \sqrt{I_{bx}^2 + I_{by}^2} \tag{10}$$

$$f_{I_b} = \tan^{-1}\left(\frac{I_{by}}{I_{bx}}\right) \tag{11}$$

In the present investigation, the performance of 10 sediment transport equations (Meyer-Peter and Müller, 1948; Einstein, 1950, Kalinske, 1947; Yalin, 1963; Engelund and Hansen, 1967; Paintal, 1971; Ackers and White, 1973; Hardisty, 1983, 1990; van Rijn, 1986; Soulsby, 1993) are examined. All formulae have been re-arranged so that their output is given in immersed weight sediment transport rate (I_b, W m^{-2}). In order to use the formulae for rippled bed conditions, the skin friction component of the shear stress (rather than the total) should be used in the computations. The Shields' criteria for the initiation of sediment movement has been adopted here.

Ripple migration
The rate of migration was measured using the ripple crests. Sediment transport rates are derived by assuming a simple relationship between the ripple migration rate and volume of bed material being transported. The immersed weight transport rate for a ripple of height η, migrating at a rate U is approximately equal to $0.5\eta U$ per unit width per unit time (e.g. Dyer, 1986).

3. Results
Bedload transport rates
Empirical formulae for the prediction of bedload transport have been derived originally for use with time-varying

unidirectional flows, such as those experienced in rivers and canals. Application of these formulae to the marine environment is complicated by the simultaneous presence of 'steady' tidal currents and oscillatory wave action. Considerable variability has been found in the application of these formulae to specific investigations (see eg Pattiaratchi and Collins, 1984; Dyer and Soulsby, 1988).

The drag effect (C_d) was evaluated by dividing the friction velocity based on the grain roughness, by the total friction velocity. The results are given in Table 1. The proportion of the skin friction component over the total shear stress was calculated. The skin friction represents 58% of the total drag on the sea bed, over the investigated area. Bedload transport rates have been predicted using (10) formulae, considering that 58% of the total drag corresponds to skin friction. Nine of the equations produced results within a band occupying 3 orders of magnitude. Results using the Ackers and White (1973) approach vary by a further 3 orders of magnitude. Ackers and White values are not considered here any further.

The ability of each of the predictive formulae to reproduce the measured transport rates is investigated now using the method of Zanke (1987). In this approach, the spread of the results about a line of 'best-fit' is evaluated:

$$r_{i=1,n} = \frac{Q_E}{Q_M} \tag{12}$$

where Q_E is the rate predicted by the formula under investigation, and Q_M is the measured transport rate determined from ripple migration rates. The 'weighting' or inaccuracy of a prediction is equal either side of the observed value. So any prediction which has a value of 1/2 the observation is given a value of twice the observed value. Although the magnitude of the error is different, the factor by which the prediction is in error, in this case by 2, is the same. Zanke (1987) formulated this as:

$$r_i = r_i \ \text{if} \ r_i < 1 \tag{13}$$

$$r_i = \frac{1}{r_i} \quad \text{if} \ r_i > 1 \tag{14}$$

All of the r_i values are multiplied together to give a final value, S:

$$S = \left[\text{Product} \left(r_i \right)_{i=n}^{i=1} \right]^{\frac{1}{n}} \tag{15}$$

Hence, the bedload transport formula with S closest to unity has performed best. The results are shown in Table 2. Considerable variation in predicted values is evident. Of the formulae investigated, that of Soulsby (1993) is found to be the most successful in reproducing the observed transport rates. However, the Zanke parameter for this formula is still around 17. It was found that the measured values can be represented most accurately by the following formula:

$$I_b = Bg \left(1 - \frac{r}{r_s} \right) \left(t_b - t_{cr} \right) U_{100} \tag{16}$$

where B=1.31 x 10^{-4} m^{-2} s^{4}.

This formula has a form similar to that suggested by Hardisty (1983), and by Williams *et al.* (1989) for gravel movements. When the transport values predicted by this formula were compared with those observed, the Zanke parameter was approximately 2.7, improving considerably on the results obtained from the 10 formulae investigated here.

The substantial variation between the output of these equations, using the same input, leads to the conclusion that within the predictive method, the choice of which transport equation to use has the greatest effect upon the rates predicted. Thus it would appear that, despite the complexity of modelling stresses in combined wave and current flows, the inaccuracy of the boundary layer models is not significant compared to the inaccuracy of the presently available sediment transport formulae. Further investigation is required to test the widespread applicability of the transport formula suggested here.

Ripples formed during periods of bedload transport persist during sub-critical flows. Higher transport rates will, therefore, be inferred (from observed ripple geometry) in the approach adopted here when, in fact, little or no transport may be occurring. The discussion undertaken here assumes that all bedload movement of sand occurs within the migration of the ripples. Sand grains are therefore assumed to be contained within the same ripple as time passes. Therefore no allowance has been made for sand 'by-passing' individual ripples ie grains jumping from one ripple crest to the next.

Another assumption inherent in the transport rates observed from the photographs is that the layer of mobile sediment is assumed to be equal to the height of the ripples. Hence, the mobile layer, or depth of mixing, has been taken as extending up to 2 cm into the bed. Shimwell (1990) measured the depth of burial to be 3.2 cm. Lees (1981), whilst investigating in shallower water depths and, therefore, greater wave height to water depth ratios, estimated the depth of mixing to be 8 cm. This shallow location, being nearer the shore would be expected to have a greater mobile layer (Bray, 1990). If the mobile layer extends below the height of the ripples, then there is sediment transport occurring outside the ability of the photographs to record it. However, in order to make the sediment transport rates inferred from the observed rates of ripple migration correlate with those predicted by many of the transport formulae investigated here, the depth of burial would have to be increased by over 2 orders of magnitude. This estimate would seem to be unlikely.

4. Conclusion

Ripple migration was observed throughout the (3 day) deployment. The rates of movement provide a means of evaluating the performance of predictive sediment bedload transport rate formulae.

A combined bed shear stress approach has been adopted to determine predicted sediment transport rates. Combined bed shear stresses have been evaluated by superimposing the instantaneous wave shear stresses, on the measured steady (unidirectional) current shear stress, over a complete wave cycle. The proportion of the total energy responsible for sediment transport (skin friction), in combination with an appropriate bedload transport formula, can provide remarkably good agreement between observed and predicted rates. The proportion of the total friction velocity that is taken up in overcoming the form drag of the ripples is within the values suggested by previous investigations.

In the present investigation, Equation 16 most accurately reproduced the transport rates inferred from ripple migration rates. This recalibration of a previous approach requires further investigation to test it's more widespread applicability.

Further investigation into the depth of the mobile sediment layer under a rippled sand bed in combined wave and current flows needs to be undertaken. Further, the velocity profile of this mobile layer needs to be established in order that improved accuracy of sediment transport rates determined from time-lapse photography of the surface of the sea bed may be obtained.

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Burst	U_{*c} (cm s^{-1})	GM82, GG83			CJ85		C_D (ave)
		U'_{*c} (cm s^{-1})	C_D		U''_{*c} (cm s^{-1})	C_D	
2	5.0	3.2	0.64		2.3	0.46	0.55
3	2.3	1.5	0.64		1.0	0.43	0.54
4	1.4	0.8	0.59		0.0	0.32	0.45
5	3.8	2.3	0.59		1.5	0.39	0.49
7	2.3	1.2	0.52		0.6	0.24	0.38
8	3.7	2.6	0.71		2.0	0.55	0.63
9	2.1	1.3	0.59		0.1	0.35	0.47
10	2.4	1.7	0.70		1.3	0.52	0.61
14	4.2	3.3	0.78		2.8	0.66	0.72
15	4.0	3.2	0.80		2.7	0.68	0.74
16	0.1	0.1	0.56		0.0	0.25	0.40
17	2.9	2.3	0.80		1.9	0.66	0.73
18	3.0	2.4	0.79		2.0	0.65	0.72
19	1.1	0.1	0.65		0.0	0.40	0.53
20	4.0	3.4	0.83		2.9	0.72	0.77
25	2.5	1.7	0.66		1.2	0.47	0.56

Table 1 Friction velocities based upon total roughness (U_{*c}) and grain roughness only (U'_{*c}), together with the corresponding drag coefficient (C_D). The drag values (C_d) for each burst represent the proportion of the total friction velocity that acts directly upon the sea bed sediment. Values have been determined for two heights (thickness) of the boundary layer.
Key: GM82- Grant and Madsen (1982); GG83- Grant and Glenn (1983); CJ85- Christoffersen and Jonsson (1985).

Sediment transport formula	Zanke parameter (S)
Yalin (1963)	17100
van Rijn (1986)	118
Meyer-Peter and Muller (1948)	365
Hardisty (1983)	651
Paintal (1971)	1720
Engelund and Hansen (1967)	118
Einstein (1950)	1190
Kalinske (1947)	554
Soulsby (1993)	17.7

Table 2 Performance of (nine) predictive equations (Ackers and White values omitted as they lie frequently at zero), assessed using the criteria of Zanke (1987) (see text). Those equations with S values nearest unity have performed best (3 significant figures).
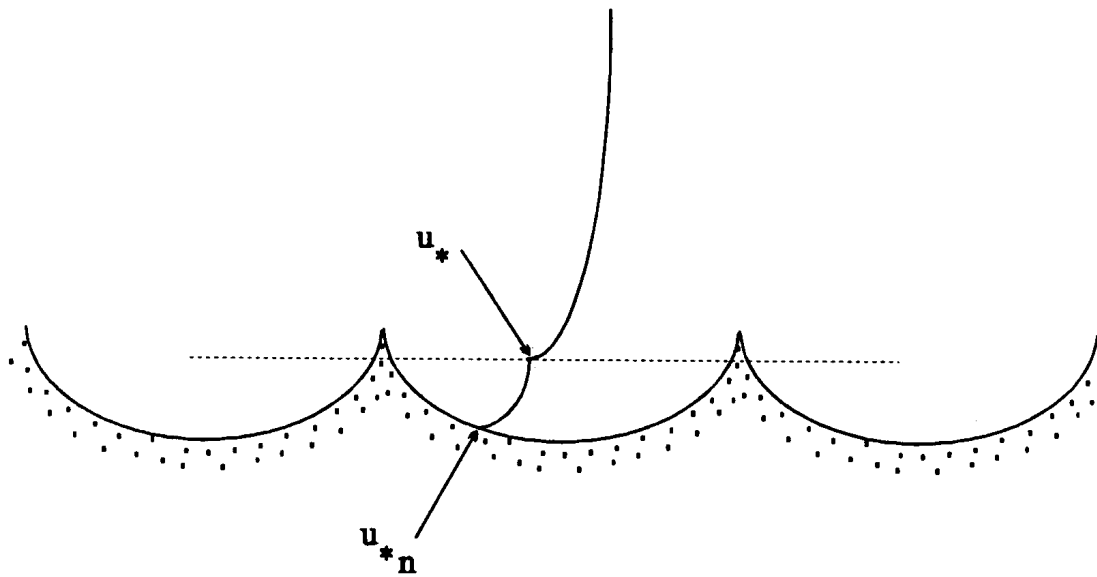

Figure 1 – Diagram showing the two-stage velocity profile used to obtain friction velocities acting on the bed sediments only.