Migration of sand waves in shallow shelf seas

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Abstract
Sand waves form a prominent regular bed pattern, which can be found on the seabed of shallow seas. The position of these sand waves is not fixed. They are assumed to migrate in the direction of the residual current. We are interested in what the physical mechanisms are, causing sand waves to migrate and at what rate. We assumed that sand waves evolve as free instabilities of the system. Within this study, a linear stability analysis has been performed on a 2DV morphological model describing the interaction between vertically varying water movement and an erodible bed in a shallow sea. Here we break the symmetry of the basic flow by choosing it as a constant current on top of a sinusoidal tidal motion. This constant current can be generated by two different physical mechanisms: by a wind stress applied at the sea surface or by a pressure gradient. As the vertical structure in these cases is different, we have investigated both. The results show that it is possible for sand waves to evolve under these flow conditions and that they are able to migrate in the direction of the residual flow. The migration aspect introduced in this paper has not been validated yet. Before we want to commence the validation, we want to gain insight into the mechanisms crucial for migration with this model.

Introduction
Large parts of shallow seas, as for instance the North Sea (Figure 1), are covered with bed features, which are fascinatingly regular. Sand waves form a prominent bed pattern having crests spaced about 500 metres apart. Their heights can lead up to several metres. They are observed at a water depth of the order of 30 metres. This means that the relative sand wave height is very significant. It is largely assumed that their crests are oriented perpendicular to the principal current [Johnson et al., 1981], [Langhorne, 1981], [Tobias, 1989]. On a theoretical basis, Hulscher [1996] found that the sand wave crests can be rotated slightly anti-clockwise with respect to the principal current, for up to 10^0.

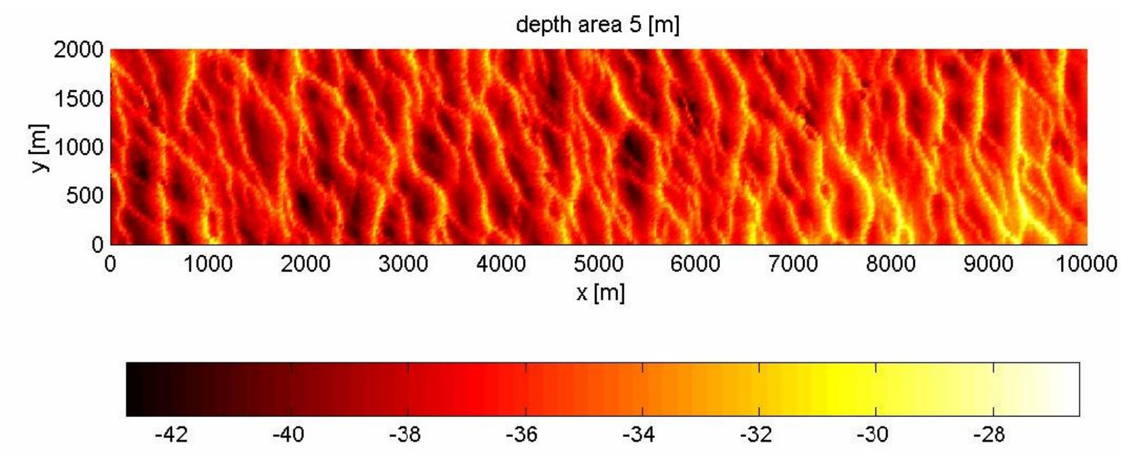

Figure 1: Bathymetry measurements made in the North Sea near the Eurogeul [Source data: North Sea Directorate; Created by: Knaapen (University of Twente)]

Migration has been investigated in the past and has already been connected to studies done into bedform dynamics in rivers. Although high velocities can be found in shallow seas like the North Sea, the long-term averaged amount of bed load is often still small. This is attributed to the fact that the residual current in a tidal environment is much smaller compared with unidirectional currents found in rivers. The migration velocities of sand waves are therefore one to two orders of magnitude smaller than the velocities attained by dunes in rivers [Allen, 1980]. Fredsøe and Deigaard [1993] described the behaviour of finite amplitude dunes under a unidirectional current. To model sand waves, they applied their dune model in a tidal environment by assuming that the time-dependence can be neglected.

Huthnance [1982] was the first to look at a system of sandbanks as being a dynamically coupled system, consisting of seawater and an erodible seabed. Within this framework, one can investigate whether certain regular patterns get excited, being free instabilities of the system. Hulscher [1996] used this principle to explain the formation of sand waves due to symmetrical tidal motion. In this model sand waves do not migrate. The predictive ability of this model for sand wave occurrence has been shown in [Hulscher and Van den Brink, 1999]. Blondeaux et al. [1999] introduced forcing due to surface waves, which changes the velocity distribution over the vertical. Migration is indicated without focussing on the mechanisms leading to this behaviour.

We can conclude that the mechanisms leading to migration of sand waves in a tidal environment are not understood yet. The research question used within this paper can be described as follows: “What are the physical mechanisms causing sand waves in shallow shelf seas to migrate and at what rate?”

In order to gain insight into the physical mechanisms involved in the migration of sand waves, an analytical model has been set up, based on the model presented by Hulscher [1996]. The model includes tidal motion together with a residual current. This residual current is assumed to be responsible for the migration of sand waves. This assumption is based on the results obtained by Fredsøe and Deigaard [1993].

First a description of the idealised model is given. It is based on the two dimensional vertical shallow water equations together with a simple sediment transport formula, describing bed load transport. Next, the method of scaling the different variables is presented. The scaling method results into the usage of two time scales (the tidal period and the morphological time scale). The morphological changes are calculated over a longer time scale than the water movement. This makes it possible to average the bottom evolution over the tidal period. Subsequently a linear stability analysis is performed. The basic state consists of a constant current, together with tidal movement (M_2). This constant current is either induced by a wind stress applied at the sea surface or induced by a pressure gradient. The initial behaviour of the system is then investigated by the feedback of small amplitude sand waves. Lastly, the results will shortly be discussed.

1. Description of the analytical model

We will start from the analytical model [Hulscher, 1996] based on the three-dimensional model used to describe sand waves and tidal sandbanks. It will be used to look at the driving factors behind migration.

1.1 Flow model

The three-dimensional study [Hulscher, 1996] has shown that the sand wave crests are oriented almost perpendicular with respect to the tidal motion. Furthermore, the Coriolis force only slightly affects sand waves. The behaviour of sand waves can therefore also be based described with the help of the two-dimensional Navier-Stokes equations. Making the shallow water approximation results in the two-dimensional shallow water equations (See also Figure 2):

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -g\frac{\partial \zeta}{\partial x} + A_v\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\right) \qquad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \qquad (2)$$

Here u and w are the velocities in respectively the x- and z- directions. t and g are respectively the time and the acceleration due to gravity. $z=\zeta$ is the free surface elevation and A_v is the vertical viscosity. The horizontal viscosity is neglected. Komarova and Hulscher [1999] have shown that this term does not introduce any new mechanism and does not change the results significantly. They furthermore showed that the shallow water approximation does not change the mechanisms or results significantly.

1.2 Boundary conditions and assumptions

The boundaries in the horizontal plane are located infinitely far away. The boundary conditions at the surface are defined as follows, whereby τ_w describes the wind induced stress at the sea surface:

$$A_v \frac{\partial u}{\partial z} = \tau_w \,;\; \frac{\partial \zeta}{\partial t} + u \frac{\partial \zeta}{\partial x} = w \tag{3}$$

The horizontal flow components at the bottom are described with the help of a partial slip condition (with S the resistance parameter). The vertical component is described by a kinematic condition:

$$A_v \frac{\partial u}{\partial z} = Su \,;\; \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} = w \tag{4}$$

1.3 Sediment transport and seabed behaviour

The sediment transport model only consists of bed load transport. This mode of transport is considered to be dominant in the tidal offshore regimes we are looking at. As the flow velocities in the vertical direction are being calculated explicitly, bed load transport can be modelled here as a direct function of the bottom shear stress. The following general bed load formula is used:

$$S_b = \alpha |\tau_b|^b \left[\tau_b - \lambda \frac{\partial h}{\partial x} \right] \tag{5}$$

The level of the seabed is denoted by h with respect to the undisturbed water depth H. S_b is the volumetric sediment transport. b is the power of transport and a a dimensionless parameter. l is the scale factor for the bed slope mechanism, taking into account that sand is transported more easily downwards than upwards. τ_b is the bottom shear stress. The effect of a critical shear stress is not explicitly studied in the model at this moment. The bed shear stress is defined as:

$$\tau_b = A_v \left(\frac{\partial u}{\partial z} \right) \Bigg|_{z=bottom} \tag{6}$$

The net inflow of sediment is assumed to be zero. This results in the following sediment balance, which couples the flow model with the sediment transport model:

$$\frac{\partial h}{\partial t} + \frac{\partial S_b}{\partial x} = 0 \tag{7}$$

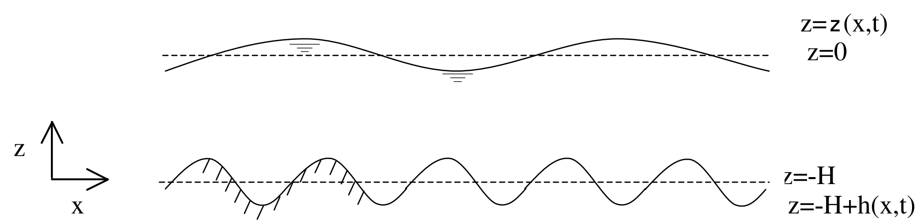

Figure 2: Illustration framework model

2. Scaled set of equations

2.1 Scaling

The variables in the equations are scaled as follows making the study of the behaviour of sand waves possible:

$$(u, w) = (Uu_*, \varepsilon U w_*)\,;\,(x, z, h) = \left(\frac{1}{\varepsilon}\delta x_*, \delta z_*, \delta h_*\right) \tag{8}$$

$$t = t_* \sigma^{-1}\,;\,\frac{UL\sigma}{g}\zeta_*\,;[\tau] = \nu_0 U / \delta \tag{9}$$

Time (t) is scaled with the tidal frequency (σ^{-1}) because tidal movement is assumed to be the main forcing mechanism of these large-scale bed forms. For the time scale of the evolution of the bottom a longer time scale is used (T_{long}). The horizontal velocity (u) is scaled with the tidal velocity amplitude (U) being in the order of 1 m s^{-1}. The vertical co-ordinate (z) and the amplitude of the bottom perturbation (h) are scaled with the Stokes depth (δ), as was done in Komarova and Hulscher [1999]. δ is an expression related to the thickness of the boundary layer:

$$\delta = \sqrt{\frac{2\nu_0}{\sigma}} \tag{10}$$

The shear stress (τ_b) is scaled with the viscosity together with the amplitude of the tidal velocity divided by the Stokes depth. This is analogous to the definition of shear stress. The water level is scaled with the length over which the tidal wave varies (L).

The last two scales to be discussed here are the scale for the vertical velocity (v) and the horizontal length scale (x). The parameter ε is introduced here making it possible to scale the variables with physical relevant scales combined with a correct order of magnitude. The value of ε (order of magnitude 10^1) is obtained by looking at the balance between the shear stress and the slope term:

$$\tau_b \approx \lambda_1 \frac{\partial h}{\partial x} \text{ and } \tau_b \approx \lambda_2 |\tau_b| \frac{\partial h}{\partial x} \tag{11}$$

With:

$$\lambda_1 = \frac{3\Theta_{c0}\, g(s-1)d}{2\tilde{\gamma}\tan\phi_s}\,;\quad \lambda_2 = \frac{1}{\tan\phi} \tag{12}$$

2.2 Set of equations

The equations describing the movement of water can now be presented in non-dimensional form

$$\frac{\partial u}{\partial t} + Ru\frac{\partial u}{\partial x} + Rw\frac{\partial u}{\partial z} = -R\frac{L\sigma}{U}\frac{\partial \zeta}{\partial x} + \frac{\partial}{\partial z}\left(E_v \frac{\partial u}{\partial z}\right) \tag{13}$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{14}$$

The boundary conditions at the free surface become:

$$\frac{\partial u}{\partial z} = \hat{\tau}_w\,;\,\frac{L\sigma^2}{g\varepsilon}\frac{\partial \zeta}{\partial t} + \frac{UL\sigma}{g\delta}u\frac{\partial \zeta}{\partial x} = w \tag{15}$$

and at the bottom:

$$E_v \frac{\partial u}{\partial z} = \hat{S}u\,;\,\frac{1}{R}\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} = w \tag{16}$$

This results furthermore in the following bottom evolution equation:

$$\frac{\partial h}{\partial T_m} = -\frac{\partial}{\partial x}\left(|\tau_b|^b \left[\tau_b - \hat{\lambda}\frac{\partial h}{\partial x}\right]\right) \tag{17}$$

The following parameters can be observed in the equations shown above (13)-(17):

$$E_v = \frac{A_v}{\sigma\delta^2};\ R = \frac{\varepsilon U}{\delta\sigma};\ \hat{S} = \frac{S}{\sigma\delta};\ \hat{\lambda} = \frac{\varepsilon\delta}{\nu_0 U}\lambda \tag{18}$$

$$T_m = \hat{\alpha}t;\ \hat{\alpha} = \alpha\frac{\varepsilon}{\sigma\delta^2}\left(\frac{\nu_0 U}{\delta}\right)^{1+b} \equiv \frac{1}{\sigma T_{long}};\ \hat{\tau}_w = \tau_w\frac{\delta}{UA_v} \tag{19}$$

E_v can be seen as a measure for the influence of the viscosity on the water movement (by definition the tidal movement) in the water column. $\hat{S}$ is a parameter describing the resistance at the bottom. R can be physically interpreted as the square root of the Reynolds number times ε.

2.3 Typical values for a North Sea location

The tidal velocity amplitude (U) is of the order 1 m s^{-1}. The viscosity (v_0 equal to A_v) are set at 0.01. The frequency of the M_2-tide in the North Sea (σ) is about 1.4 10^{-4} s^{-1}. The proportionality constant α can be computed from Van Rijn. It is set at a value of about 0.3 sm^{-2}. The downhill sediment transport coefficient (λ) was set at two and b is set at 1/2. The average depth (H) is in the order of 30 metres [Hulscher, 1996]. For a wind speed *W* the wind-induced stress at the sea surface can be approximated by [Prandle, 1997]:

$$\tau_w = 0.013W^2 \tag{20}$$

3. Linear stability analysis

The solution of the problem can formally be presented by the vector $\psi = (u,w,\zeta,h)$. Starting from an exact solution of the problem, a certain basic state ψ_0 can be perturbed by a small amplitude ($\gamma < 1$) perturbation. The solution can be expanded as follows:

$$\psi = \psi_0 + \gamma\psi_1 + \gamma^2\psi_2 + \gamma^3\psi_3 + \dots \tag{21}$$

For $\gamma < 1$ and $\| \psi_1 \| = \| \psi_i \|$ ▪(1) the successive terms decrease in magnitude. This means that the one but largest contribution is fully given by the linear term $\gamma\psi_1$. Therefore, the instability of the basic state ψ_0 can be tested by determining the initial behaviour of ψ_1. Amplification of ψ_1 in time means that ψ_0 is unstable and decay means that ψ_0 is stable.

3.1 Basic state

The basic state describes a tidal current together with a constant current over a flat bottom (horizontally uniform flow). A Taylor expansion in the parameter γ enables transferring the boundary condition from $z = -1 + h$ to $z = -1$. The following set of equations arises:

$$\frac{\partial u_0}{\partial t} = -R\frac{L\sigma}{U}\frac{\partial \zeta_0}{\partial x} + \frac{\partial}{\partial z}\left(E_v\frac{\partial u_0}{\partial z}\right);\ \frac{\partial w_0}{\partial z} = 0 \tag{22}$$

With the boundary condition at the free surface:

$$\frac{\partial u_0}{\partial z} = \hat{\tau}_w \tag{23}$$

And the boundary conditions at the bottom:

$$E_v\frac{\partial u_0}{\partial z} = \hat{S}u_0;\ w_0 = 0 \tag{24}$$

The velocity in the horizontal direction for the basic state can be formulated as follows:

$$u_0 = u_r(z) + u_s(z)\sin t + u_c(z)\cos t \tag{25}$$

Firstly, a description for the constant current ($u_r(z)$) is derived. Two possible types of constant currents will be discussed here. These are a wind driven (I) current and a current induced by a pressure gradient (II):

Wind stress (I)	**Pressure gradient (II)**
$u_r = \hat{\tau}_w\left(1+\frac{E_v}{S}+z\right)$	$u_r = P\left(\frac{1}{2}z^2-\frac{E_v}{S}-\frac{1}{2}\right)$

With:

$$P=\frac{\varepsilon L}{\delta E_v}\zeta \tag{26}$$

The shear stress at the bottom is therefore equal to the wind stress in case (I) and equal to P ($1\cdot10^5$ times the pressure gradient) in case (II). In figure 3 the velocity distribution over the vertical (0 denotes the sea surface and –1 the sea bed) can be found for the wind and pressure gradient induced currents. They are set at a depth-averaged velocity of 0.1 m s^{-1} in the figure. The velocities at the seabed are not zero in this case and in the case of tidal movement. Models, which consist of a z-independent viscosity formulation, overestimate the bottom shear stress. The shear stress determines directly the amount of sediment transported. Therefore, the slip parameter S needs to be finite, if the model is to produce realistic results.

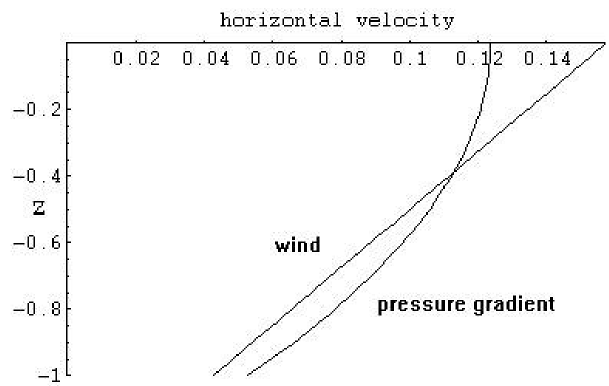

Figure 3: velocity distribution over the vertical for a wind stress and pressure induced current

Tidal movement (with an amplitude of 1 m s^{-1}) can be described by (See (25)) [Hulscher, 1996]:

$$u_s(z)=-\operatorname{Im}(\hat{u}(z));\ u_c(z)=-\operatorname{Re}(\hat{u}(z)) \tag{27}$$

$$\hat{u}(z)=-\frac{i}{2}\alpha^{-2}\left(\frac{\cosh(\alpha z)-\cosh(\alpha)-\frac{\alpha E_v}{\hat{S}}\sinh\alpha}{\left(\frac{1}{\alpha}-\frac{\alpha E_v}{\hat{S}}\right)\sinh\alpha-\cosh\alpha}\right) \tag{28}$$

$$\alpha=(1+i)\sqrt{\frac{1}{E_v}} \tag{29}$$

3.2 Perturbed state

The stability of the basic state can be tested by determining the initial behaviour of ψ_1. Subtracting the basic state (22)-(24) and the smaller order terms leaves:

$$\frac{\partial u_1}{\partial t}+Ru_0\frac{\partial u_1}{\partial x}+Rw_1\frac{\partial u_0}{\partial z}=-R\frac{L\sigma}{U}\frac{\partial \zeta_1}{\partial x}+\frac{\partial}{\partial z}\left(E_v\frac{\partial u_1}{\partial z}\right) \tag{30}$$

$$\frac{\partial u_1}{\partial x}+\frac{\partial w_1}{\partial z}=0 \tag{31}$$

The boundary conditions at the free surface are

$$\frac{\partial u_1}{\partial z} = w_1 \tag{32}$$

and at the bottom:

$$\frac{\partial u_1}{\partial z} = \frac{\hat{S}}{E_v} u_1 + h_1 \frac{\hat{S}}{E_v} \frac{\partial u_0}{\partial z} - h_1 \frac{\partial^2 u_0}{\partial z^2} \tag{33}$$

$$w_1 = u_0 \frac{\partial h_1}{\partial x} \tag{34}$$

The unknowns are Fourier transformed as follows with $\psi_1 = (u_1, w_1, \zeta_1, h_1)$ [Hulscher, 1996]:

$$\psi_1 = \iint \tilde{\psi}(t) e^{-ikx} dk + c.c. \tag{35}$$

in which c.c. means complex conjugate and k is the wave number of the wavy bottom perturbation. Harmonic truncation in time is applied. This means that the perturbation is restricted to a finite number of tidal components. In the case of a unidirectional tide, the following truncation will contain the dominant physical processes [Hulscher, 1996].

$$\tilde{u}_{trunc}(z,t) = \tilde{h}\left[ia_0(z) + a_s(z)\sin t + a_c(z)\cos t\right] \tag{36}$$

$$\tilde{w}_{trunc}(z,t) = \tilde{h}\left[c_0(z) + ic_s(z)\sin t + ic_c(z)\cos t\right] \tag{37}$$

$$\tilde{\zeta}_{trunc}(t) = \tilde{h}\left[d_0 + id_s \sin t + id_c \cos t\right] \tag{38}$$

The bottom variable will hardly vary on a tidal time scale. The evolution of the seabed can therefore at best be described by averaging the sediment fluxes over the tidal period. The solution for the bottom evolution equation can now be written with h_0 the initial small amplitude and ω describing the behaviour of the bed:

$$\tilde{h} = h_0 e^{\omega_r T_m} \cos(kx - \omega_i T_m) \tag{40}$$

The real and imaginary part of ω can be written as follows, with the brackets denoting the tidal average:

$$\omega_r = -k(b+1)a_0'(-1)\left\langle |\tau_{b0}|^b \right\rangle + k^2 \hat{\lambda} \left\langle |\tau_{b0}|^b \right\rangle \tag{41}$$

$$\omega_i = -k(b+1)a_s'(-1)\left\langle |\tau_{b0}|^b \sin t \right\rangle - k(b+1)a_c'(-1)\left\langle |\tau_{b0}|^b \cos t \right\rangle \tag{42}$$

The Fourier transformed functions can now be solved numerically together with the boundary conditions. The shear stress can then be calculated, which is then to be imported in the bottom evolution equation.

4. Results

The obtained solution is complex. The real and imaginary parts represent respectively the initial growth and migration rates of the sand waves. As was found in previous research positive growth rates appear for various wavelengths. These are taken into account by the real part of ω. This means that sand waves can evolve due to the interaction between water movement and seabed morphology.

Furthermore, migration of the sand waves has been found. These displacements of the bottom perturbations are described by the imaginary part of ω. They can be considered as phase shifts of the sinuses describing the sand waves. These phase shifts are caused by the introduced asymmetry in the water movement. They depend on the nature and magnitude of the asymmetry in the water movement. If the constant current is set to zero, the water movement is symmetrical. In this case, no migration is found since the time average of the imaginary part (between brackets) is then zero.

An estimate of the order of magnitude of the migration velocity of the sand waves can be deduced from ω_i. A typical wavelength for a sand wave is 500 meters. The morphological time scale (T_m) is about 6 years.
A basic state is taken consisting of tidal movement (M_2) with an amplitude of 1 m s^{-1} (based on the results from [Hulscher, 1996]) together with a wind driven current with a depth averaged velocity of 0.1 m s^{-1}. With this basic state an estimate of the term within brackets can be obtained. The first term has a value of 0.2 and the second one is two orders of magnitude smaller. $a_s(-1)$ is determined based on the results obtained by Hulscher [1996] and has an order of magnitude of 10^0. The second term with $a_c(-1)$ is also much smaller. The second term can therefore be left out of this first estimate. A migration rate in the order of about 6 meters per year is now obtained. In the case of a pressure gradient induced constant current with a depth averaged velocity of 0.1 m s^{-1}, the shear stress would be smaller at the seabed. This would then result in a slightly smaller migration rate of the sand waves.

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