The influence of sediment transport formulae on modelling river dune development

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ABSTRACT: Simulating river dunes, with the eventual goal to predict dune dimensions and propagation speed in the field, offers many challenges. In this study we simulated river dunes in a 2DV numerical dune development model and evaluated the effect of different processes in bed load transport on the dune. This is done based on the Meyer-Peter & Müller transport formula, with two different adjustments. First an increased critical shear stress was used and secondly spatial relaxation was implemented. The results of this study show that the balance between the bed shear stress and the critical shear stress, is more important to obtain correct trends for dune height for low flow than for high flows. Parameterizing these processes needs to be done with care.

1 INTRODUCTION

In heavily navigated rivers, such as the Rhine, dredging to maintain a prescribed navigable depth is of utmost economic relevance. This fairway maintenance focusses on reducing bottle necks caused by local shoals. One of the sources of these shoals are river dunes, often propagating over already shallow areas in the river.

Predictions of river dune propagation and height can support efficient planning of maintenance dredging, or estimations of the shallowest points in the fairway. To determine both dune height and propagation for such predictions, numerical dune development models can be used. These types of models vary from models based on the 2D Saint-Venant equations in the vertical plane (Giri & Shimizu, 2006; Paarlberg et al., 2009; Nelson et al., 2011) to fully 3D models using direct numerical simulation (Nabi et al., 2013). The main constraint for predictions is that the calculation times of the predicted variables should be shorter than real time, preferably by at least 1 to 2 orders of magnitude smaller.

A model that has been validated for dune development with small calculation times is the dune development model by Paarlberg et al. (2009). While this model has been validated on flume studies representing high flow regimes (Paarlberg et al., 2009) and has been applied for the transition to upper stage plane bed (van Duin et al., 2017, 2021), it still needs validation for low flow regimes. As during low flows shoals caused by dunes may hamper shipping most.

Sediment transport during low flows is assumed to be bed load dominant. In the Waal River, dunes during low flows decrease in height but increase in length (Lokin et al., 2022). A similar pattern is visible for dunes during extreme high flow in the transition towards upper stage plane bed (Naqshband et al., 2017). However, during these low flows the underlying mechanism is different.

Therefore, the objective of this study was to investigate the influence of different bed load dominant sediment transport mechanisms, included in the Meyer-Peter & Müller (1948) transport equation. on dune development during low to median flow.

2 MODEL DESCRIPTION

The dune development model used in this study, is the model developed by (Paarlberg et al., 2009). This model was built on work for marine sand waves (Hulscher, 1996; Németh et al., 2006; van den Berg et al., 2012). Further model development has been focussing on (extreme) high flows and the transition to upper stage plane bed (Van Duin & Hulscher, 2014; van Duin et al., 2021) and dune splitting (Warmink et al., 2014). In this section, the model set-up as used in this study is shortly introduced.

2.1 Flow

The flow is described by the 2D Saint-Venant equations in the vertical plane. The most important assumption within the flow module is the turbulence closure, which is solved through a constant eddy viscosity. The vertical boundary conditions are periodic. This simulates a virtually infinite row of identical dunes, by only one dune in the model domain.

For a full description of the equations and the numerical implementation we refer to the work by Paarlberg et al. (2009) and van den Berg et al. (2012).

2.2 Sediment transport and bed update

Sediment transport is based on the Meyer-Peter & Müller (1948) formula (Eq. 1). This formula is based on bed load transport and through the critical shear stress incipient motion is taken into account. Because of the relatively steep slopes found in dunes, the shear stresses are corrected for the slope (e.g. Sekine & Parker, 1992). The total bed load transport is determined by the following set of equations:

$$q_b = \begin{cases} \beta[\tau-\tau_c]^n\left[1+\eta\frac{\partial z_b}{\partial x}\right] & \text{if } \tau-\tau_c \geq 0 \\ 0 & \text{if } \tau-\tau_c < 0 \end{cases} \qquad (1)$$

$$\tau_c = \tau_{c,0}\,\frac{1+\eta\frac{\partial z_b}{\partial x}}{\sqrt{1+\left(\frac{\partial z_b}{\partial x}\right)^2}} \qquad (2)$$

$$\tau_{c,0} = \tau_c^* g \Delta D_{50} \qquad (3)$$

where β is the calibration factor determined by $\beta = \frac{m}{\Delta g}$ with m is 4 and Δ the relative density of sand normalised by the density of the water, τ is the local shear stress, τ_c the critical shear stress, and n is the non-linearity factor set to 3. The term $\left[1+\eta\frac{\partial z_b}{\partial x}\right]$ is related to the bed slope correction, with η is th bes slope parameter which is inversely related to the angle of repose ϕ and $\frac{\partial z_b}{\partial x}$ the local bed slope. Equation 2 shows the bed slope correction on the critical shear stress, whith $\tau_{c,0}$ as the non-corrected critical shear stress, which is dependent on the critical Shields parameter (τ_c^*) and the median grain size (D_{50}) (Eq. 3).

Optionally, the Meyer-Peter & Müller formula can be extended by a spatial relaxation formula (Equation 4). This extension was proposed by Tsujimoto et al., (1990) and is based on the assumption that sediment transport needs some distance to adapt to spatially changing flow conditions.

$$\frac{\partial q_b}{\partial x} = \frac{q_{b,e}-q_b}{\Lambda} \qquad (4)$$

where $q_{b,e}$ is the equilibrium transport determined by Equation 1, q_b is the actual transport, and Λ is the step length (Einstein, 1950)and is defined by:

$$\Lambda = \alpha D_{50} \qquad (5)$$

with α is the non-dimensional step length, determined following (van Duin et al., 2021)

The sediment tranport as determined based on the flow field is translated into a bed update using the Exner equation, with a correction for the bed porosity (ε):

$$(1-\varepsilon)\frac{\partial z_b}{\partial t} = -\frac{\partial q_s}{\partial x} \qquad (6)$$

3 RESEARCH METHOD

3.1 Validation data

To validate the model results, the dune height and propagation speed are compared to these parameters as found in the Waal River (Netherlands). The derivation and the exact values of these parameters can be found in Lokin et al. (2022). While the aim of this study is to assess the influence of the

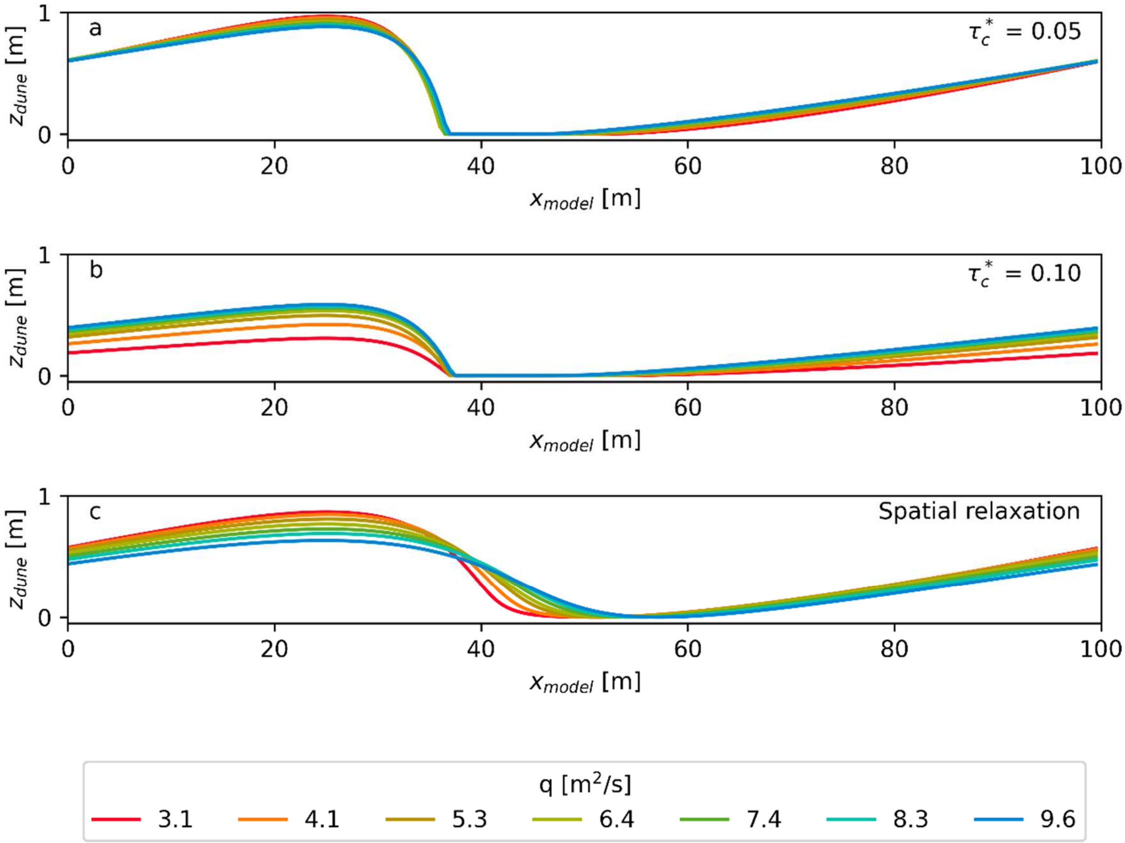

Figure 1. Simulated dune shapes for the different sediment transport conditions. The vertical axis represents z_{dune}, which is the bed level with respect to the trough. a) Normal Shields stress conditions b) Increased Shields stress conditions and c) normal shields stress with spatial relaxation conditions.

sediment transport equation, the trends in these data are more important than the exact values. Therefore, the model is not calibrated to fit perfectly to the data.

3.2 Simulations and model settings

Flow conditions and sediment sizes are in line with the conditions in the Waal river (Lokin et al. submitted.). With these conditions three different variants of the Meyer-Peter & Müller equation are tested. The first condition is the “basic” formula with the bed slope correction and a critical Shield stress of $\tau_c^* = 0.05$. This is a value representative for the median grain size. The second condition has an increased critical shields stress, $\tau_c^* = 0.1$, which can be seen as the effect of a lower flow velocity due to (partial) flow separation at the lee slope. The third condition has the normal critical Shield stress combined with the spatial relaxation process.

Table 1: General simulation settings

Parameter	Value	
Domain length (L)	100	m
Specific discharge (q)	3.1 – 9.6	m²/s
Bed slope (i)	5e^{-5}	m/m
Median grain size (D_{50})	1.0	mm

The hydraulic conditions mimic the flow conditions varying from low to median conditions in the Waal River, while for all conditions the domain length is kept constant. The model settings are shown in Table 1. All simulations are run for 100 days, which ensures that dunes have reached their equilibrium shape.

4 RESULTS

4.1 Dune shape and height

Each individual simulation, with the different transport conditions, results in plausible dune shapes (Fig. 1); the simulated shapes can be found in rivers and for each simulation the lee slope angles are smaller than the angle of repose. However, the trends with regard to the specific discharge and

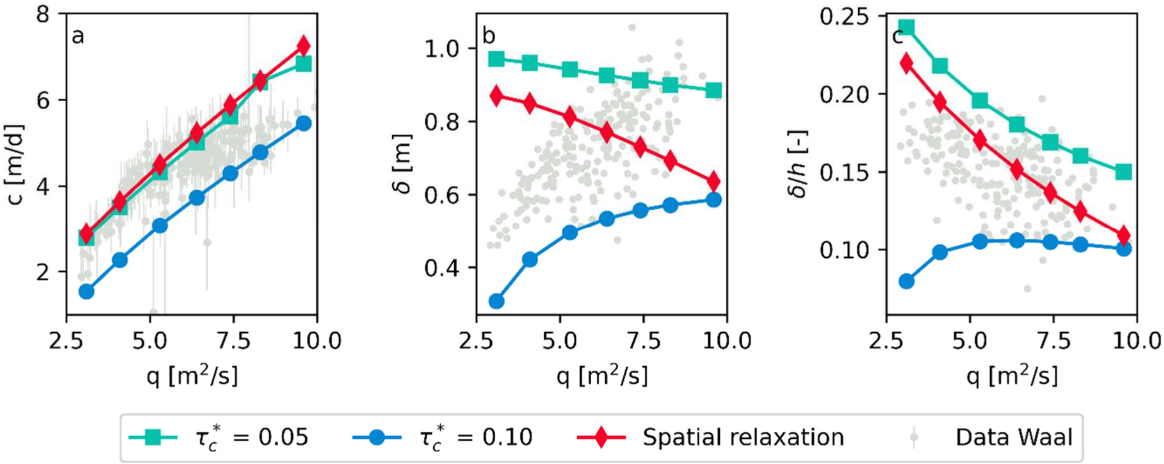

Figure 2. Propagation speed (a), dune height (b) and dune height over water depth (c) as function of the discharge for the three sediment transport variants. The light grey dots in the background are the observed values from the Waal River (Lokin et al. 2022)

therefore the flow conditions show distinctive differences between the three transport conditions.

The dunes simulated with the "normal" critical shear stress all have the same shape (Fig. 1a) and the dune height decreases by 10 cm between the lowest and highest discharges (Fig. 2b). While an increase of dune height is expected, since the water depth increases as well.

The increase of dune height from low to high discharge is found in the simulations with the increased critical Shields stress (Fig. 1b). While the rate of increase in height decreases with the increasing discharge (Fig. 2b). For the low discharges the transport capacity, mainly depending on $\tau - \tau_c$, determines the dune height. While at higher discharges this parameter appears to be of less importance.

For the dune height over water depth (Fig. 2c), the trend in the field data is best represented by the normal shields stress and linear relaxation simulations. Combined with the trend of the dune height only, it can be deducted that the simulated water depth is not corresponding to the water depth found in the Waal. Calibration of this water depth may improve these trends and therefore predictive power of the model.

4.2 Propagation speed

The propagation speed of all dunes are fairly similar to the propagation speed found in the field data (Fig. 2a). Decreasing the critical Shields stress, decreases the propagation speed accordingly.

Adding spatial relaxation barely influences the propagation speed. Because the dune height is smaller than without spatial relaxation and the total sediment transport is the same in both cases, not all sediment transport in the spatial relaxation case contributes to dune propagation. This is an artefact of the processes which are parameterized by adding this spatial relaxation; suspended sediment.

5 DISCUSSION

5.1 The effect of the critical shear stress

The results show that realistic dunes can be simulated with different variants of the Meyer-Peter & Müller sediment transport formula. However, the effect of increasing the shear stress on the dune is height opposite to the effect of adding spatial relaxation.

Increasing the critical shear stress can mimic or correct for several processes that influence the total sediment transport, which are not explicitly included in the dune development model. A first processes is linked to the turbulence closure model. Because of the sudden flow expansion at the lee side of the dune, partial flow separation occurs. This results in energy losses which are not resolved by the constant eddy viscosity. A second process is linked to hiding and exposure. The model only uses the median grain size to calculate sediment transport, while the Waal river is bi-modal (Ylla Arbós et al., 2021). Hiding and exposure increases the shear stress needed to bring sediment in motion.

Both these processes result in a smaller transport capacity, ensuing a smaller dune height and smaller propagation speeds. For the case with $q = 3.1$ m2/s, this is most significant as here the dune height is limited by the transport capacity. The dune crest can only move due to a fairly small transport capacity, where most sediment is then deposited in the dune crest. A small transport capacity can only maintain small dunes.

Increasing the discharge, thus flow velocity will lead to higher transport capacity on the stoss slope and the dune crest, while also the dune trough deepens. Eventually the dune height will reach a balance where the dune troughs will have no transport and the crest enough to maintain the equilibrium dune height and to propagate steadily.

5.2 The effect of spatial relaxation

De dune height decreases with increasing discharge for the simulations with spatial relaxation is opposite to what is expected from the field data. While spatial relaxation mimics the effect of bed material that is brought in suspension and settles further downstream, for example, on the stoss slope of the downstream dune. This is a process that is linked to the transition to upper stage plane bed (Naqshband et al., 2017). And considering the trends in dune height it may not be relevant for dune formation during low flows.

Whilst spatial relaxation is necessary to simulate the transition to upper stage plane bed, it does not contribute to low dunes during low flows. Therefore, when trying to simulate dunes over the entire discharge domain, the parameterization of this spatial relaxation needs to be revisited. Such that for both low and extreme high flow the processes that are dominant in dune formation, are also dominant in the simulation.

When simulating dunes during flood waves and extreme low flows, the parameterization of all processes that are dominant in the development in the dunes need to be revisited. Such that during low flows dune heights increase with the discharge, to decrease again at the discharge that where upper stage plane bed may be expected.

6 CONCLUSIONS

In this study we compared the effect of an increased critical shear stress and spatial relaxation implemented in the Meyer-Peter & Müller transport formula on simulated dune shapes. Processes mimicked by the increased critical shear stress, decreased flow velocities due to flow separation and hiding and exposure, result in a realistic trend for the relation dune height to discharge, for low flow cases. Spatial relaxation parameterizes the processes important for the transition to upper stage plane bed. To further improve the dune development model for forecasting, these processes need to be parameterized with great care, to ensure the correct processes are dominant in the dune development.

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